SYED AMMAL ENGINEERING COLLEGE

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DIGITAL COMMUNICATION

GATE EXAM MODEL QUESTIONS AND ANSWERS

Amplitude Modulated Systems

(a) Modulation

1. In commercial TV transmission in India, picture and speech signals are modulated respectively

(Picture)		(Speech)
(a) VSB	and	VSB
(b) VSB	and	SSB
(c) VSB	and	FM
(d) FM	and	VSB

[GATE 1990: 2 Marks]

Soln. Note that VSB modulation is the clever compromise between SSB and DSB. Since TV bandwidth is large so VSB is used for picture transmission. Also, FM is the best option for speech because of better noise immunity

Option (c)

2. In a double side-band (DSB) full carrier AM transmission system, if the modulation index is doubled, then the ratio of total sideband power to the carrier power increases by a factor of________.

[GATE 2014: 1 Mark]

Soln. The AM system is Double side band (DSB) with full carrier. The expression for total power in such modulation signal is

$$\boldsymbol{P_t = \frac{E_c{}^2}{2R} + \frac{\mu^2}{4}\frac{E_c{}^2}{2R} + \frac{\mu^2}{4}\frac{E_c{}^2}{2R}}$$

$$or,\ P_t = P_C + \frac{\mu^2}{2} P_C$$

The second term on the right hand side is side band power.

$$so, \qquad P_{SB} = \frac{\mu^2}{2} P_C$$

$$or, \qquad \frac{P_{SB}}{P_C} = \frac{\mu^2}{2}$$

Now if μ (modulation index) is doubled then $\frac{P_{SB}}{P_C}$ will be 4 times

So, it is factor of 4

Ans. Factor of 4

3. The maximum power efficiency of an AM modulator is

   (a) 25% (c) 33%

   (b) 50% (d) 100%

[GATE 1992: 2 Marks]

Soln. Efficiency of modulation can be given as

$$\eta = \frac{P_S}{P_C + P_S} = \frac{\mu^2/2\, P_C}{P_C + \frac{\mu^2}{2} P_C}$$

$$\frac{\mu^2/2}{1 + \mu^2/2} = \frac{\mu^2}{(2 + \mu^2)}$$

μ=1 is the optimum value

$$so, \eta = \frac{1}{2+1} = \frac{1}{3} \times 100 = 33\%$$

Option (c)

4. Consider sinusoidal modulation in an AM systems. Assuming no over modulation , the modulation index (μ) when the maximum and minimum values of the envelope, respectively, are 3V and 1V is ________

[GATE 2014: 1 Mark]

Soln. As given is the problem the modulation is sinusoidal this is also called tone modulation.

There is no over modulation means that modulation index is less than or equal to 1.

In such case the formula for modulation index is given by

$$\mu = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

Where E_{max} is the maximum value of the envelope

E_{min} is the minimum value of the envelope.

This method is popular when the modulated waveform is observed is CRO

$$\mu = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2} = 0.50$$

Modulation index is 0.50

5. Which of the following analog modulation scheme requires the minimum transmitted power and minimum channel band-width?

(a) VSB (c) SSB

(b) DSB-SC (d) AM

[GATE: 2005 1 Mark]

Soln.

Modulation type	**BW**	**Power**
Conventional AM	**2 f_m**	**Maximum power**
DSB SC	**2 f_m**	**(Less power)**
VSB	**f_m + vestige**	
SSB	**f_m**	**Less & power**

So, SSB least power & bandwidth

Option (c)

6. Suppose that the modulating signal is $m(t) = 2\cos(2\pi f_m t)$ and the carrier signal is $x_C(t) = A_C \cos(2\pi f_c t)$.Which one of the following is a conventional AM signal without over-modulation?

(a) $x(t) = A_C m(t)\cos(2\pi f_c t)$

(b) $x(t) = A_C[1 + m(t)]\cos(2\pi f_c t)$

(c) $x(t) = A_C \cos(2\pi f_c t) + \frac{A_C}{4} m(t)\cos(2\pi f_c t)$

(d) $x(t) = A_C \cos(2\pi f_m t)\cos(2\pi f_c t) + A_C \sin(2\pi f_m t)\sin(2\pi f_c t)$

[GATE 2010: 1 Mark]

Soln. Given

Modulation signal $\boldsymbol{m(t) = 2\cos(2\pi f_m t)}$

Carrier signal $\boldsymbol{x_c(t) = A_C \cos(2\pi f_c) t}$

Note that conventional AM is DSB – FC (DSB full carrier)

Standard Expression is given by

$e(t) = E_C[1 + m(t)] \cos \omega_c t$

Or $e(t) = E_C[1 + \mu \cos \omega_m t] \cos \omega_c t - - - - - (1)$

Option (b) is $x(t) = A_C[1 + 2\cos(2\pi f_m t)] \cos 2\pi f_c t$

Comparing this expression with the standard one given equation (I)

We get μ = 2 i.e. conventional AM with over modulation

Option (c)

$x(t) = A_c \cos 2\pi f_c t + \frac{A_C}{4} m(t) \cos 2\pi f_c t$

$= A_C [1 + \frac{1}{4}. 2\cos(2\pi f_m t) \cos 2\pi f_c t]$

$$= A_C [1 + \frac{1}{2}\cos(2\pi f_m t)] \cos 2\pi f_c t$$

Here $\mu = {}^{1}/_{2}$

So, this represents conventional AM without over modulation.

Option (d) is non standard expression

So, correct option is option (c)

7. For a message signal $m(t) = \cos(2\pi f_c t)$ and carrier of frequency f_c. Which of the following represents a single side-band (SSB) signal?
(a) $\cos(2\pi f_m t) \cos(2\pi f_c t)$
(b) $\cos(2\pi f_c t)$
(c) $\cos[2\pi(f_c + f_m)t]$
(d) $[1 + \cos(2\pi f_m t)]. \cos(2\pi f_c t)$

[GATE 2009: 1 Mark]

Soln. Option (a) in the problem represents AM signal DSB-SC. If will have both side bands

option (b) represents only the carrier frequency

Option (c), $\cos[2\pi(f_c + f_m)t]$ represents upper side band (SSB-SC). It represent SSB signal

Option (d) represents the conventional AM signal

Ans. Option (c)

8. A DSB-SC signal is generated using the carrier $\cos(\omega_C t + \theta)$ and modulating signal x(t). The envelop of the DSB-SC signal is
   (a) $x(t)$
   (b)$|x(t)|$
   (c) Only positive portion of x(t)
   (d) $x(t)\cos\theta$

[GATE 1998: 1 Mark]

Soln. Given

Carrier $c(t) = \cos(\omega_c t + \theta)$

Modulating signal $m(t) = x(t)$

DSB SC modulated signal is given by $c(t).\, m(t) = s(t)$

$$= x(t)\cos(\omega_c t + \theta)$$

$$= x(t)\{\cos\theta \,.\, \cos\omega_c t - \sin\theta \sin\omega_c t\}$$

$$= x(t)\cos\theta \,.\, \cos\omega_c t - x(t).\, \sin\theta \sin\omega_c t$$

Envelope of $s(t) = \sqrt{[x(t)\cos\theta]^2 + [x(t)\sin\theta]^2}$

$$= \sqrt{x^2(t)(cos^2\theta + sin^2\theta)}$$

$$= x(t)$$

Option (b) $|x(t)|$

9. A 1 MHz sinusoidal carrier is amplitude modulated by a symmetrical square wave of period 100 μsec. Which of the following frequencies will not be present in the modulated signal?

   (a) 990 kHz
   (b) 1010 kHz
   (c) 1020 kHz
   (d) 1030 kHz

[GATE 2002: 1 Mark]

Soln. Frequency of carrier signal is $1MHz = 1000\ KHz$

Modulation signal is square wave of period 100 μS.

Frequency $= \frac{1}{100\times10^{-6}} = 10\ KHz$

Since modulation signal is symmetrical square wave it will contain only odd harmonics i.e. 10 KHz, 30 KHz, 50 KHz -----etc.

Thus the modulated signal has

$$f_c \pm f_m = (1000 \pm 10KHz) = 1010KHz \ \& \ 990\ KHz$$

$$f_c \pm 3f_m = (1000 \pm 30KHz) = 1030KHz \ \& \ 970\ KHz$$

So, 1020 KHz will not be present in modulated signal

Option (c)

10. A message signal given by $m(t) = (\frac{1}{2})\cos\omega_1 t - (\frac{1}{2})\sin\omega_2 t$ is amplitude modulated with a carrier of frequency ω_c to generate
$s(t) = [1 + m(t)]\cos\omega_c t$
What is the power efficiency achieved by this modulation scheme?

   (a) 8.33%
   (b) 11.11%
   (c) 20%
   (d) 25%

[GATE 2009: 2 Marks]

Soln. Given

$$m(t) = \frac{1}{2}\cos\omega_1 t - \frac{1}{2}\sin\omega_2 t$$

$$s(t) = [1 + m(t)]\cos\omega_c t$$

Note that the modulation frequency are $\omega_1 and\ \omega_2$ i.e. multitone modulation

Net modulation index is $\mu = \sqrt{\mu_1^2 + \mu_2^2 + -- \mu_n^2}$

Here, $\mu = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$

$$\eta = \frac{\mu^2}{\mu^2 + 2} \times 100\%$$

$$= \frac{(1/\sqrt{2})^2}{(1/\sqrt{2})^2 + 2} \times 100\% = 20\%$$

Option (c)

11. A 4 GHz carrier is DSB-SC modulated by a low-pass message signal with maximum frequency of 2 MHz. The resultant signal is to be ideally sampled. The minimum frequency of the sampling impulse train should be

(a) 4 MHz
(b) 8 MHz
(c) 8 GHz
(d) 8.004 GHz

[GATE: 1990 2 Mark]

Soln. Given

$f_c = 4\ GHz = 4000\ MHz$

$f_m = 2\ MHz\ (low\ pass\ message\ signal)$

Such a signal is amplitude modulated (DSB-SC) i.e. two side bands $(f_c + f_m) \& (f_c - f_m)$

i.e. 4002 & 3998 or 4 MHz = BW

so, min. sampling frequency should be (Nyquist Rate)

option (b) $\boldsymbol{f_{s(min)} = 2 \times 4 = 8\ MHz}$

(b) Demodulation

12. Consider the amplitude modulated (AM) signal $A_C \cos \omega_c t + 2 \cos \omega_m t \cos \omega_c t$. For demodulating the signal using envelope detector, the minimum value of A_C should be

(a) 2 (c) 0.5
(b) 1 (d) 0

[GATE 2008: 1 Mark]

Soln. Modulated signal is given as

$$\boldsymbol{\varphi_{AM}(t) = A_c \cos \omega_c t + 2 \cos \omega_m t.\cos \omega_c t}$$

$$\boldsymbol{\varphi_{AM}(t) = [A_c + 2 \cos \omega_c t] \cos \omega_m t}$$

Note that for envelope detection the modulation should not go beyond full modulation i.e. $\boldsymbol{\mu = 1}$**, so amplitude of baseband signal has to be less than the carrier amplitude (**$\boldsymbol{A_c}$**)**

$$\boldsymbol{|f(t)|_{max} \le A_c}$$

i.e. $\boldsymbol{|2 \cos \omega_m t|_{max} = 2 \le A_c}$

or $\boldsymbol{A_c \ge 2}$

option (a)

13. Which of the following demodulator (s) can be used for demodulating the signal

$x(t) = 5(1 + 2\cos 200\,\pi t) cos 20000\pi t$

(a) Envelope demodulator
(b) Square-law demodulator
(c) Synchronous demodulator
(d) None of the above

[GATE 1993: 2 Marks]

Soln. The modulated signal given is $x(t) = 5(1 + 2\cos 200\pi t).\cos 2000\pi t$

The standard equation for AM is

$$X_{AM}(t) = A_c(1 + \mu\cos\omega_m t)\cos\omega_c t$$

If we compare the two equation we find $\mu = 2$.

The modulation index is more than 1 here, so it is the case of over modulation.

When modulation index is more than 1 (over modulation) then detection is possible only with, Synchronous modulation, such signal can not be detected with envelope detector.

Option (c)

14. The amplitude modulated wave form $s(t) = A_C[1 + K_a m(t)]\cos\omega_C t$ is fed to an ideal envelope detector. The maximum magnitude of $K_a m(t)$ is greater than 1. Which of the following could be the detector output ?

(a) $A_c m(t)$
(b) $A_c^2[1 + K_a m(t)]^2$
(c) $|A_C[1 + K_a m(t)]|$
(d) $A_C[1 + K_a m(t)]^2$

[GATE 2000: 1 Mark]

Soln. Given

$|K_a m(t)| > |$

For the above condition the AM signal is over modulated. Envelope detector will not be able to detect over modulated signal correctly.

Non of the above options

15. The diagonal clipping in Amplitude Demodulation (using envelope detector) can be avoided if RC time-constant of the envelope detector satisfies the following condition, (here W is message bandwidth and ω is carrier frequency both in rad/sec)

(a) $RC < \frac{1}{W}$

(b) $RC > \frac{1}{W}$

(c) $RC < \frac{1}{\omega}$

(d) $RC > \frac{1}{\omega}$

[GATE 2006: 2 Marks]

Soln. It is seen that to avoid negative peak clipping also said diagonal clipping the RC time constant of detector should be

Or $\boldsymbol{\tau < \frac{1}{f_m}}$

Note f_m is maximum modulating frequency i.e. the bandwidth w

So, $\boldsymbol{RC < \frac{1}{w}}$

16. An AM signal is detected using an envelope detector. The carrier frequency and modulation signal frequency are 1 MHz and 2 KHz respectively. An appropriate value for the time constant of the envelope detector is

(a) 500 μsec

(b) 20 μsec

(c) 0.2 μsec

(d) 1 μsec

[GATE 2004: 1 Mark]

Soln. Note that the time constant RC should satisfy the following condition

$$\boldsymbol{\frac{1}{f_c} < RC < \frac{1}{f_m}}$$

$$\boldsymbol{\frac{1}{1\times10^6} < RC < \frac{1}{2\times10^3}}$$

Or $\boldsymbol{1\,\mu s < RC < 0.\ 5ms}$

Option (b)

17. A DSB-SC signal is to be generated with a carrier frequency $f_c = 1MHz$ using a non-linear device with the input-output characteristic

$V_0 = a_0 v_i + a_1 v^3{}_i$

Where a_0 and a_1 are constants. The output of the non-linear device can be filtered by an appropriate band-pass filter.

Let $V_i = A^i_c \cos(2\pi f^i_c t) + m(t)$ where m(t) is the message signal. Then the value of f^i_c (in MHz) is

(a) 1.0
(b) 0.333
(c) 0.5
(d) 3.0

[GATE 2003: 2 Marks]

Soln. $\boldsymbol{V_0 = \varphi_0[A^i_c \cos(2\pi f^i_c t) + m(t)]}$

$$\boldsymbol{+\varphi_1[A^i_c \varphi_{AM}(t) = A_c \cos \omega_c t\ 2 \cos \omega_c t]}$$

$$\boldsymbol{= \varphi_0[A^i_c \cos(2\pi f^i_c t) + m(t)]}$$

$$\boldsymbol{+\varphi_1[(A^i_c)^3 cos^3(2\pi f^i_c t) + m^3(t)]}$$

$$\boldsymbol{+3.\ A^i_c \cos(2\pi f^i_c t).\ m^2(t) + 3.\ (A^i_c)^2 cos^2(2\pi f^i_c t).\ m(t)}$$

AM – DSB – SC signal lies is

$\boldsymbol{\varphi_1.\ 3(A^i_c)^2 m(t) cos^2(\pi f^i_c t)}$

For DSB – SC the last term is important

$$\boldsymbol{3\varphi_1 (A^i_c)^2 cos^2 2\pi f^i_c t.\ m(t)}$$

$$\boldsymbol{3\ \varphi_1 (A^i_c)^2.\ m(t).\ [1 + \cos 2\pi(2f^i_c)t]}$$

Note $\boldsymbol{m(t) \cos \omega_c t \rightarrow f_c (carrier) = 1MHz}$

For $\boldsymbol{cos^2}$ term as expended the term is having $\boldsymbol{2f^i_c}$

$\boldsymbol{2f^i_c = 1MHz \quad so,\ f^i_c = 0.5\ MHz}$

Option (c)

18. A message signal $m(t) = \cos 2000\,\pi t + 4\cos 4000\,\pi t$ modulates the carriers $c(t) = \cos 2\pi f_c t$ where $f_c = 1MHz$ to produce an AM signal. For demodulating the generated AM signal using an envelope detector, the time constant RC of the detector circuit should satisfy

(a) 0.5 ms< RC < 1 ms

(b) 1 μs << RC < 0.5 ms

(c) RC<< 1 μs

(d) RC >> 0.5 ms

[GATE 2011: 2 Marks]

Soln. Message signal is

$$\boldsymbol{m(t) = \cos 2000\pi t + 4\cos 4000\pi t}$$

It consist of two frequencies $\omega_1 = 2000\pi$

Or $2\pi f_1 = 2000\pi$

Or $f_1 = 1KHz$

$$f_2 = 2\ KHz$$

So, Max frequency is 2 KHz

So, $\frac{1}{f_c} << RC < \frac{1}{f_m}$

$$\frac{1}{1\ MHz} << RC < \frac{1}{2\ KHz}$$

Or, $1\mu s << RS < 0.\ 5ms$

Option (b)

(c) *Receivers*

19. A super heterodyne radio receiver with an intermediate frequency of 455 KHz is tuned to a station operating at 1200 KHz. The associated image frequency is ------- KHz

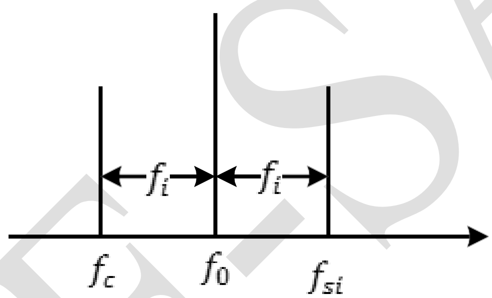
[GATE 1993: 2 Marks]

Soln. In most receivers the local oscillator frequency is higher than incoming signal i.e.

$$\boldsymbol{f_0 (frequency\ of\ local\ oscillator) = f_s + f_i}$$

Where f_s ....signal frequency

$f_i or\ f_{si}$..............Image frequency



$$\boldsymbol{f_{si} = f_s + 2IF = f_s + 2f_i}$$

$$\boldsymbol{f_{si} = 1200 + 2(455)}$$

$$\boldsymbol{f_{si} = 2110\ KHz}$$

so, answer is 2110 KHz

20. The image channel selectivity of superheterodyne receiver depends upon

(a) IF amplifiers only
(b) RF and IF amplifiers only
(c) Pre selector, RF and IF amplifiers
(d) Pre selector and RF amplifiers

[GATE 1998: 1 Marks]

Soln. Image rejection depends on front end selectivity of receiver and must be achieved before If stage. So image channel selectivity depends upon pre selector & RF amplifier. If it enters IF stage it becomes impossible to remove it from wanted signal.

Option (d)

NOTE:- Similar problems have appeared in GATE 1995, 1996 and 1987. Only problem statement is different otherwise they are same problems.

Digital Modulation Schemes

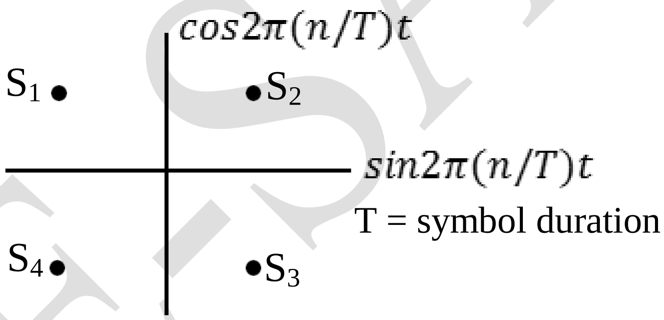
1. In binary data transmission DPSK is preferred to PSK because
   (a) a coherent carrier is not required to be generated at the receiver
   (b) for a given energy per bit, the probability of error is less
   (c) the 180^0 phase shifts of the carrier are unimportant
   (d) more protection is provided against impulse noise

[GATE 1989: 2 Marks]

Soln. Differential phase shift (DPSK) is non coherent version of the PSK. It is differentially coherent modulation method. DPSK does not need synchronous (Coherent) carrier at the demodulator. The input sequence of binary bits is modified such that the next bit depends upon the previous bit

Option (a)

2. For the signal constellation shown in the figure, the type of modulation is



[GATE 1991: 2 Marks]

Soln. The given constellation has four signals which are 90^0 apart with the adjacent signal

These waveforms correspond to phase shifts of 0^0, 90^0, 180^0 and 270^0 as shown in the phase diagram.

The type of modulation is QPSK

3. Quadrature multiplexing is
   (a) the same as FDM
   (b) the same as TDM
   (c) a combination of FDM and TDM
   (d) quite different from FDM and TDM

[GATE 1998: 1 Mark]

Soln. Quadrature carrier multiplexing (QCM) enables two DSBSC modulated waves, resulting from two different message signals to occupy the same transmission bandwidth and two message signals can be separated at the receiver.

It is also called Quadrature Amplitude Modulation (QAM) so it is quite different from FDM and TDM

Option (d)

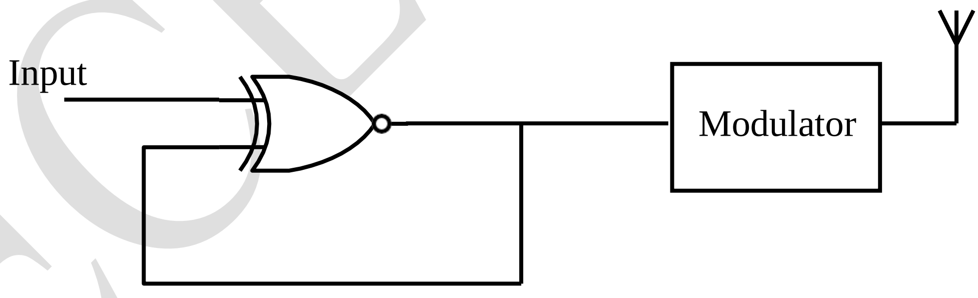
4. The message bit sequence to a DPSK modulator is 1,1,0,0,1,1 . The carrier phase during the reception of the first two message bits is π, π. The carrier phase for the remaining four message bits is
(a) π, π,0, π
(b) 0,0, π, π
(c) 0, π, π, π
(d) π, π,0,0
[GATE 1998: 2 Marks]

Soln. Message bits sequence 1 1 0 0 1 1

Let, Logic 1 → 0^0

Logic 0 → π

Ref. bit = 0



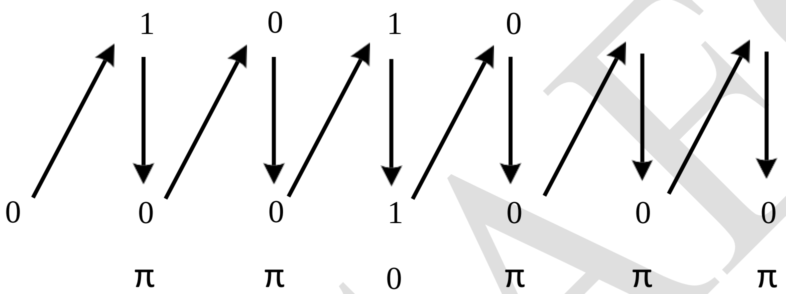
Ex NOR

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

$Y = A \odot B$

$= AB + \bar{A}\bar{B}$

(output is 1 when both input are same)



Remaining message bits are

0 π π π

Option (c)

5. The bit steam 01001 is differentially encoded using ‘Delay and Ex OR’ scheme for DPSK transmission. Assuming the reference bit as a ‘1’ and assigning phases of ‘0’ and ‘π’ for 1’s and 0’s respectively, in the encoded sequence, the transmitted phase sequence becomes
(a) π 0 π π 0
(b) 0 π π 0 0
(c) 0 π π π 0
(d) π π 0 π π

[GATE 1992: 2 Marks]

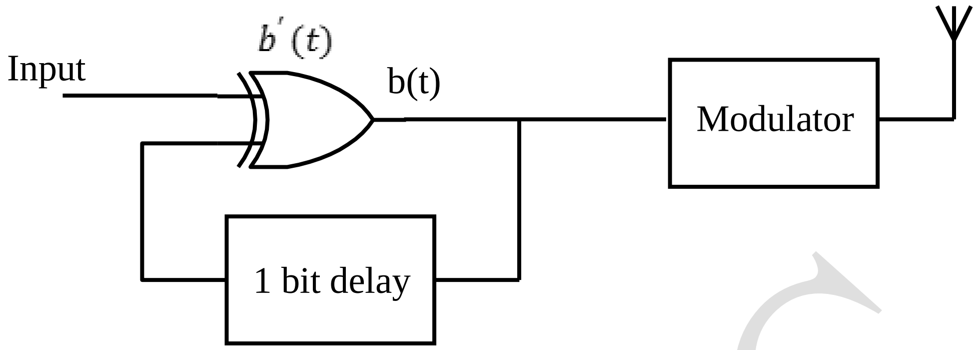
Soln.

EX-OR

A	B	Y
0	0	0
0	1	1
1	0	1
0	0	0

$Y = A \oplus B$

Output is 1 when both input are different



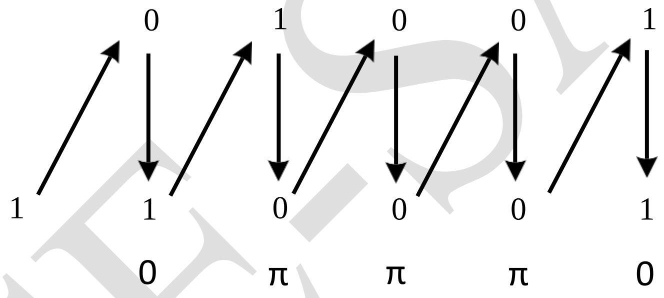
Bit stream

0 1 0 0 1

Ref. bit 1

Logic 0 $\rightarrow \pi$

Logic 1 $\rightarrow$ 0



Option (c)

6. A video transmission system transmits 625 picture frames per second. Each frame consists of a 400 x 400 pixel grid with 64 intensity levels per pixel. The data rate of the system is

(a) 16 Mbps
(b) 100 Mbps
(c) 600 Mbps
(d) 6.4 Gbps

[GATE 2001: 2 Marks]

Soln. Frames per sec = 625

Pixels per frame $= 400 \times 400$

64 intensity levels per pixels

Can be represented by bits per pixel

Data rate $= 625 \times 400 \times 400 \times 6 = 600\ Mbps$

Option (c)

7. The bit rate of digital communication system is R kbit/s. The modulation used is 32-QAM. The minimum bandwidth required for ISI free transmission is
(a) R/10 Hz
(b) R/10 KHz
(c) R/5 Hz
(d) R/5 KHz

[GATE 2013: 1 Mark]

Soln. In an ideal Nyquist channel, bandwidth required for ISI (Inter Symbol Interference) free transmission is

$$W = \frac{R_b}{2}$$

Here modulations is 32 QAM

i.e. $2^n = 32\ or\ n = 5\ bits$

Signaling rate is

$$R_b = \frac{R}{5} kbps$$

Where R is bit rate

Min. bandwidth

$$W = \frac{R_b}{2} = \frac{R}{5 \times 2} = \frac{R}{10} KHz$$

Option (b)

8. For a bit-rate of 8 kbps, the best possible values of the transmitted frequencies in a coherent binary FSK system are
(a) 16 KHz and 20 KHz
(b) 20 KHz and 32 KHz
(c) 20 KHz and 40 KHz
(d) 32 KHz and 40 KHz

[GATE 2002: 1 Mark]

Soln. Given

Bit rate = 8 kbps

The transmitted frequencies in coherent BFSK should be integral multiple of 8 i.e. the option

32 KHz & 40 KHz is the choice.

Since both frequencies are multiple of 8

Option (d)

9. An M-level PSK modulation scheme is used to transmit independent binary digits over a band-pass channel with bandwidth 100 KHz. The bit rate is 200 kbps and the system characteristic is a raised-cosine spectrum with 100% excess bandwidth. The minimum value of M is ________

[GATE 2014: 2 Marks]

Soln. Bandwidth $(B) = \frac{R_b}{log_2 M}(1+\alpha)$

Or, $100\ KHz = \frac{200\times10^3}{log_2 M}$

Or, $log_2 M = 4$

Or, M = 16

10. In a baseband communication link, frequencies upto 3500 Hz are used for signaling. Using a raised cosine pulse with 75% excess bandwidth and for no inter symbol interference, the maximum possible signaling rate is symbols per sec is

(a) 1750
(b) 2625
(c) 4000
(d) 5250

[GATE 2012: 1 Mark]

Soln. For raised cosine spectrum transmission bandwidth is given as

$$B_T = \frac{R_b}{2}(1+\alpha)$$

Where α – Roll off factor

R_b – bit rate

$$B_T = \frac{R_b}{2}(1+\alpha)$$

Where R_b – maximum signaling rate

Or, $3500\ Hz = \frac{R_b}{2}(1 + 0.75)$

Or,

$$R_b = \frac{3500 \times 2}{1.75} = 4000\ bps$$

Option (c)

11. Coherent orthogonal binary FSK modulation is used to transmit two equiprobable symbol waveforms $s_1(t) = \alpha \cos 2\pi f_1 t$ and $s_2(t) =$ acos $2\pi f_2 t,$ where $\alpha\ is\ 4\ mV$. Assume an AWGN channel with two-sided noise power spectral density $\frac{N0}{2} = 0.5 \times 10^{-12}\ W/Hz$. Using an optimal receiver and the relation.

$$Q(v) = \frac{1}{\sqrt{2\pi}} \int_v^{\infty} e^{-\frac{u^2}{2}} du, \quad \text{the bit error probability}$$

For a data rate of 5000 kbps is

(a) $Q(2)$
(b) $Q(2\sqrt{2})$
(c) $Q(4)$
(d) $Q(4\sqrt{2})$

[GATE 2014: 2 Marks]

Soln. For coherent FSK modulation probability of error

$$(P_e) = \frac{1}{2}\ erfc\left[\frac{E_b}{2N_0}\right]$$

Given

Data rate $R_b = 500\ kbps = 500 \times 10^3\ bps$

$$\frac{N_0}{2} = 0.5 \times 10^{-12}\ W/\ Hz$$

$$\alpha = 4 \times 10^{-3} V$$

$$T_b = \frac{1}{Rb} = \frac{1}{500 \times 10^3} = 2 \times 10^{-6}\ sec$$

Signal energy $E_b = T_b \times signal\ Power$

$$= 2 \times 10^{-6} \times \frac{A^2}{2}$$

$$= 2 \times 10^{-6} \times \frac{\alpha^2}{2}$$

$$= 2 \times 10^{-6} \times \frac{(4\times10^{-3})^2}{2}$$

$$= 10^{-6} \times 16 \times 16^{-6}$$

$$E_b = 16 \times 12^{-12} \text{ Joules}$$

$$Here \quad P_e = \frac{1}{2} erfc\left[\sqrt{\frac{16 \times 12^{-12}}{2 \times 10^{-12}}}\right]$$

$$= \frac{1}{2}\ erfc\ [\sqrt{8}] = \frac{1}{2}\ erfc\ \left[\frac{4}{\sqrt{2}}\right]$$

Note,

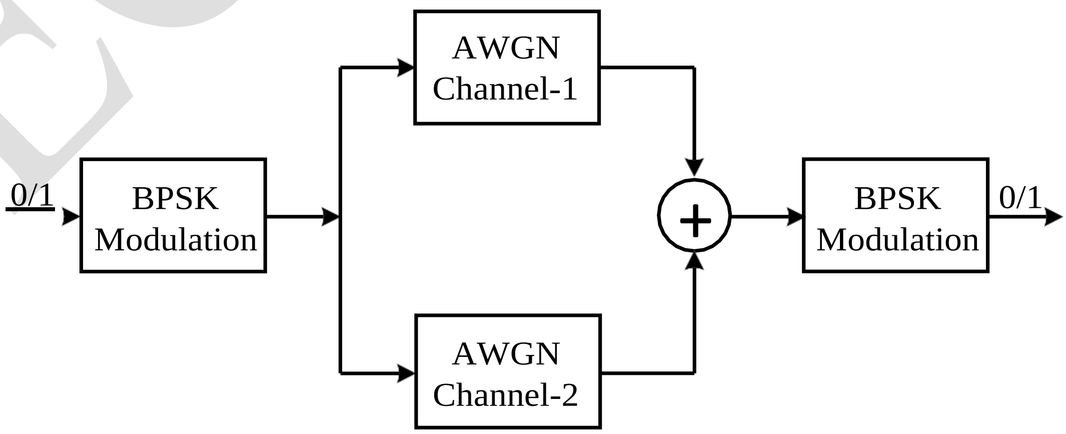
$$\frac{1}{2}\, erfc\left[\frac{y}{\sqrt{2}}\right] = Q(y)$$

$$P_e = Q(4)$$

Option (c)

12.Let $Q(\sqrt{\gamma})$ be the BER of a BPSK system over an AWGN channel with two-sided noise power spectral density $N_0/2$. The parameter γ is a function of bit energy and noise power spectral density.

A system with two independent and identical AWGN channels with noise power spectral density $N_0/2$ is shown in the figure. The BPSK demodulator receives the sum of outputs of both the channels.



If the BER of this system is $Q(b\sqrt{\gamma})$, then the value of b is________

[GATE 2014: 2 Marks]

Soln. Given,

Bit error rate (BER) of BPSK system with AWGN channel $= Q\sqrt{\gamma}$
additive white Gaussian Noise (AWGN) with power spectral density $N_0/2$
γ parameter is function of bit energy and Noise power spectral density.

Demodulator receives the output of both channels.

$$So, BER = Q(\sqrt{\gamma + \gamma})$$

$$= Q(\sqrt{2\gamma})$$

$$= Q(\sqrt{2}\sqrt{\gamma})$$

By comparing we find

$$Q(\sqrt{2}\sqrt{\gamma}) = Q(b\sqrt{\gamma})$$

$$So, b = \sqrt{2}$$

13. A BPSK scheme operating over an AWGN channel with noise power spectral density of $N_0/2$, uses equiprobable signals

$$S_1(t) = \sqrt{\frac{2E}{T}\sin(\omega_c t)} \quad and\ S_2(t) = \sqrt{\frac{2E}{T}\cancel{\sin}(\omega_c t)}$$

Over the symbol interval (0, T). If the local oscillator in a coherent receiver is ahead in phase by 45^0 with respect to the received signal, the probability of error in the resulting system is

(a) $Q\left(\sqrt{\frac{2E}{N_0}}\right)$

(b) $Q\left(\sqrt{\frac{E}{N_0}}\right)$

(c) $Q\left(\sqrt{\frac{E}{2N_0}}\right)$

(d) $Q\left(\sqrt{\frac{E}{4N_0}}\right)$

[GATE 2012: 2 Marks]

Soln. We know that the probability of error in coherent BPSK is given by

$$P_e = Q\left[\sqrt{\frac{2E}{N_0}}\right]$$

Since the local oscillator in coherent receiver is ahead by 45^0 with respect to received signal. It will decrease the signal energy by factor of

$cos^2 45^0 = \frac{1}{2}$

$So,\quad P_e = Q\left[\sqrt{\frac{E}{N_0}}\right]$

Option (b)

14. At a given probability of error, binary coherent FSK is inferior to binary coherent PSK by.

(a) 6 dB
(b) 3 dB
(c) 2 dB
(d) 0 dB

[GATE 2003: 2 Marks]

Soln. Probability of error for coherent PSK and FSK is given as

For FSK

$$P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

PSK

$$P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

From the table of error function table it is found that Binary FSK is 3 dB inferior to binary PSK

Option(b)

15. The input to a matched filter is given by

$$s(t) = \begin{cases} 10\sin(2\pi \times 10^6) & 0 < t < 10^{-4} sec \\ 0 & Otherwise \end{cases}$$

The peak amplitude of the filter output is

(a) 10 volts
(b) 5 volts
(c) 10 millivolts
(d) 5 millivolts

[GATE 1999: 2 Marks]

Soln. In digital modulation schemes the function of receiver is to distinguish between two transmitted signals in presence of noise. Receiver is said to be optimum if it yields minimum probability of error. It is called matched filter when noise at receiver is white. Matched filter can be implemented as integrate and dump correlation receiver.

Maximum amplitude of matched filter output is

$$\frac{A^2 T_b}{2} = \frac{10^2}{2} \times 10^{-4} = 5\ mV$$

16. Coherent demodulation of FSK signal can be detected using
(a) correlation receiver
(b) band pass filters and envelope detectors
(c) matched filter
(d) discriminator detection

[GATE 1992: 2 Marks]

Soln. For coherent detection one can use matched filter or correlation receiver, others are not coherent. Matched filter is used when you have only one signal. But FSK has two signals of different frequencies

So, use Correlation receiver

Option (a)

17. In a BPSK signal detector, the local oscillator has a fixed phase error of 20^0. This phase error deteriorates the SNR at the output by a factor of
(a) $\cos 20^0$
(b) $\cos^2 20^0$
(c) $\cos 70^0$
(d) $\cos^2 70^0$

[GATE 1990: 2 Marks]

Soln. In BPSK if detector has fixed phase error, say ϕ, then output power would change by a factor of $\cos^2\phi$

So, option (b)

Pulse Modulation Systems

(a) Sampling

1. A bandlimited signal is sampled at the Nyquist rate. The signal can be recovered by passing the samples through
   (a) an RC filter
   (b) an envelope detector
   (c) a PLL
   (d) an ideal low-pass filter with the appropriate bandwidth

[GATE 2001: 1 Mark]

Soln. A continuous time signal is sampled at Nyquist rate

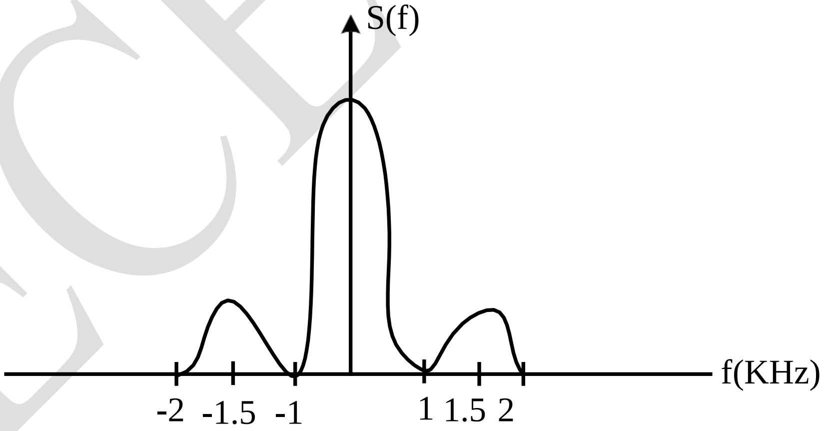
i.e. $f_s = 2f_m$ Samples/sec

It can be fully recovered.

The spectrum F(ω) repeats periodically without overlap. The signal can be recovered by passing sampled signal through a low pass filter with sharp cut- off at frequency f_m.

Option (d)

2. A deterministic signal has the power spectrum given in figure. The minimum sampling rate needed to completely represent signal is



(a) 1 KHz
(b) 2 KHz
(c) 3 KHz
(d) none of these

[GATE 1997: 1 Mark]

Soln. The given power spectrum has main lobe and other side lobes. We observe that the maximum frequency of main lobe is 1KHz.

practically about 90% of the total signal lies in the major lobe. So maximum frequency of the signal can be taken as 1 KHz.

Minimum sampling rate (Nyquist rate) = $2f_m$

Nyquist rate = $2 \times 1KHz = 2KHz$

Option (b)

3. A signal has frequency components from 300 Hz to 1.8 KHz. The minimum possible rate at which the signal has to be sampled is ---

[GATE 1991: 2 Marks]

Soln. This is the case of band pass sampling

$$f_H = 1800\ Hz$$

$$f_L = 300\ Hz$$

Bandwidth $= f_H - f_L = 1800 - 300 = 1500\ Hz$

Now, $m = \frac{f_H}{BW} = \frac{1800}{1500} = 1.2 \quad , Integer = 1.0$

$$(f_s)_{min} = \frac{2f_H}{m} = \frac{2\times1800}{1} = 3600\ samples/sec$$

Ans. 3600 samples/sec

4. Flat top sampling of low pass signals
   (a) gives rise to aperture effect
   (b) implies oversampling
   (c) leads to aliasing
   (d) introducing delay distortion

[GATE 1998: 1 Mark]

Soln. Flat top sampling of low pass signal has the spectrum of sinc function where amplitude of high frequency components is reduced. This affect is called Aperture effect. An aperture refers to the sampling process as window (aperture) of finite time width through which signal voltage is observed.

This can be reduced by either increasing the sampling frequency or reducing sampling aperture width. As width increases loss increases

Option (a)

5. Increased pulse width in the flat top sampling leads to
   (a) attenuation of high frequencies in reproduction
   (b) attenuation of low frequencies in reproduction
   (c) greater aliasing errors in reproduction
   (d) no harmful effects in reproduction

[GATE 1994: 1 Mark]

Soln. For flat top sampling the spectrum is of sinc function, where the amplitude of high frequency component is reduced. This effect is called aperture effect.

As the sampling aperture increases i.e. increase of pulse width ------ there will be more loss in high frequency components.

Option (a)

6. A 1.0 KHz signal is flat top sampled at the rate of 1800 samples/sec and the samples are applied to an ideal rectangular LPF with cut-off frequency of 1100 Hz, then the output of the filter contains
   (a) only 800 Hz component
   (b) 800 Hz and 900 Hz components
   (c) 800 Hz and 1000 Hz components
   (d) 800 Hz, 900 Hz and 100 Hz components

[GATE 1995: 1 Mark]

Soln. Given

$$\boldsymbol{f_m = 1KHz}$$

$$\boldsymbol{f_s = 1.8\ k\ samples/sec}$$

The frequency components in the sampled signal are

$$\boldsymbol{nf_s \pm f_m}$$

For $\boldsymbol{n = 0, \quad f_m = 1KHz = 1000Hz}$

$\boldsymbol{n = 1, \quad frequencies\ are\ 1.8 \pm 1\ i.e.\ 800Hz\ \&\ 2800\ Hz}$

$n = 2, \quad frequencies\ are\ 3.6 \pm 1\ i.e.\ 2600Hz\ \&\ 4600\ Hz$

Cutoff frequency of LPF 1100 Hz

So, 800 Hz & 1000 Hz components will appear at the output

Option (c)

7. The Nyquist sampling interval, for the signal $\sin c(700\,t) + \sin c(500\,t)$ is

(a) $\frac{1}{350}\,sec$ $\qquad$ $\frac{1}{700}\,sec$

(b) $\frac{\pi}{350}\,sec$ $\qquad$ (d) $\frac{\pi}{175}\,sec$

(c)

[GATE 2001: 2 Marks]

Soln. Signal is given as

$$\sin c\,(700t) + \sin c\,(500t)$$

$$= \frac{\sin(700\pi t)}{700\pi t} + \frac{\sin(500\pi t)}{500\pi t}$$

The maximum frequency component is

$$2\pi f_m = 700\pi$$

Or $f_m = 350Hz$

The Nyquist rate is $f_s = 2f_m$

$$= 2 \times 350\ Hz$$

$$= 700\ Hz$$

Sampling interval $\left(\frac{1}{f_s}\right) = \frac{1}{700}\,sec$

Option (c)

8. The Nyquist sampling frequency (in Hz) of a signal given by $6 \times 10^4 \sin c^2(400t) * 10^6 \sin c^3(100t)$ is

(a) 200 $\qquad$ (c) 1500

(b) 300 $\qquad$ (d) 1000

[GATE 1999: 2 Marks]

Soln. **The signal given is**

$$6 \times 10^4 \sin c^2\,(400t) * 10^6 \sin c^3\,(100t)$$

Let the given function is f(t) and

$f_1(t) = 6 \times 10^4 \sin c^2(400t)$

$f_2(t) = 10^6 \sin c^3\,(100t)$

We know that

$$f_1(t) * f_2(t) \rightleftharpoons F_1(\omega).\,F_2(\omega)$$

Convolution in time domains is product is frequency domain

Bandwidth of

$F_1(\omega) = 2 \times 400\ rad/sec = 800\ rad/\sec\ or\ 400\ Hz$

Due to $\sin c^2$ term

Bandwidth of

$F\,_2(\omega) = 3 \times 100 = 300\ rad/\sec\ or\ 150Hz$

Due to $\sin c^2$ term

Then $F_1(\omega).\,F_2(\omega)$ will have bandwidth of 150Hz only (since product of two spectrums)

Sampling frequency is $2 \times 150Hz = 300Hz$

Option (b)

9. Four independent massages have bandwidths of 100 Hz. 100 Hz, 200 Hz and 400 Hz, respectively. Each is sampled at the Nyquist rate, and the samples are Time Division Multiplexed (TDM) and transmitted. The transmitted rate (in Hz) is

(a) 1600
(b) 800
(c) 400
(d) 200

[GATE 1999: 2 Marks]

Soln. Four messages are sampled at the Nyquist rate

I – message $f_{s1} = 200Hz$

II – message $f_{s2} = 200Hz$

III – message $f_{s3} = 400Hz$

IV – message $f_{s4} = 800Hz$

So there are 1600 samples in 1 sec

So, the speed of commutator is 1600 samples per sec

Option (a)

10. A signal $x(t) = 100\cos(24\pi \times 10^3)\, t$ is ideally sampled with a sampling period of 50 µsec and then passed through an ideal low pass filter with cutoff frequency of 15 KHz. Which of the following frequency is/are per present at the filter output?
    (a) 12 KHz only
    (b) 8 KHz only
    (c) 12 KHz and 9 KHz
    (d) 12 KHz and 8 KHz

[GATE 2002: 2 Marks]

Soln. Signal frequency $\frac{24\pi\times10^3}{2\pi} = 12\ KHz$

Sampling period $T_S = 50\ \mu sec$

So $f_s = \frac{1}{T_S} = \frac{1}{50\times10^{-6}} = 20\ KHz$

After sampling, signal will have frequency f_m and $f_s \pm f_m$

i.e. 12 KHz and 20 KHz – 12 KHz = 8 KHz

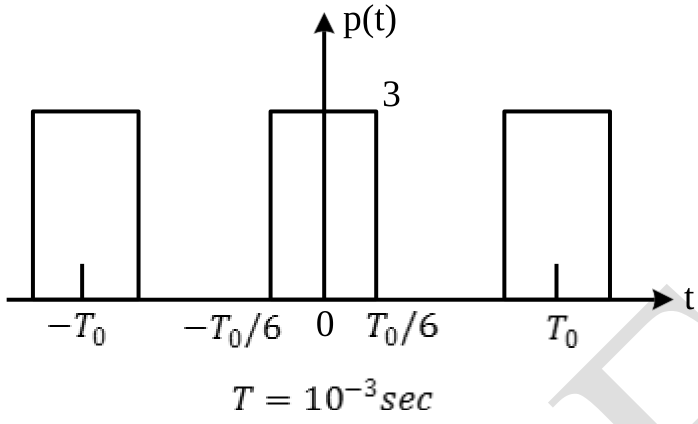
so, after filter the frequency present will be 12 KHz and 8 KHz

Option (d)

Note that sampling frequency (20 KHz) is less than 2 f_m (24 KHz) Nyquist rate, so there will be aliasing error.

11. Let $x(t) = 2\cos(800\pi t) + \cos(1400\pi t)$. $x(t)$ is sampled with the rectangular pulse train shown in the figure. The only spectral components

(in KHz) present in the sampled signal in the frequency range 2.5 KHz to 3.5 KHz are



(a) 2.7, 3.4
(b) 3.3, 3.6
(c) 2.6, 2.7, 3.3, 3.4, 3.6
(d) 2.7, 3.3

[GATE 2003: 2 Marks]

Soln. Given $T = 10^{-3}$ so frequency $= \frac{1}{10^{-3}} = 1\ KHz$ of pulse train.

To determine frequency components in pulse train we find Fourier series Coefficient C_n

$$C_n = \frac{1}{T_0}\int_{-T_0/6}^{T_0/6} A e^{-jn\omega_0 t}\, dt = \frac{1}{T_0}\cdot\frac{A}{(-jn\omega_0)}\left[e^{-jn\omega_0 t}\right]_{-T_0/6}^{T_0/6}$$

$$= \frac{A}{T_0(-jn\omega_0)}\left[e^{-jn\omega_0 T_0/6} - e^{jn\omega_0 T_0/6}\right]$$

$$C_n = \frac{A.2.j}{T_0(jn\omega_0)}\sin\left(\frac{n\pi}{3}\right) = \frac{A}{n\pi}\sin\left(\frac{n\pi}{3}\right)$$

From C_n we see that harmonics present are 1,2,3,4,5,7--------

So p(t) has 1 KHz, 2 KHz, 4 KHz-------

The signal has frequency components 0.4 KHz and 0.7 KHz

The sampled signal will be $x(t) \times p(t)$ will have

1 ± 0.4 and 1 ± 0.7 KHz

2 ± 0.4 and 2 ± 0.7 KHz

3 ± 0.4 and 3 ± 0.7 KHz etc

In the frequency range of 2.5 KHz to 3.5 KHz

The frequency present are

2 + 0.7 = 2.7 KHz

4 – 0.7 = 3.3 KHz

Option (d)

12. The Nyquist sampling rate for the signal $s(t) = \frac{\sin(500\pi t)}{\pi t} \times \frac{\sin(700\pi t)}{\pi t}$ is given by

(a) 400 Hz
(b) 600 Hz
(c) 1200 Hz
(d) 1400 Hz

[GATE 2010: 2 Marks]

Soln. Given

$$s(t) = \frac{\sin(500\pi t)}{\pi t} \times \frac{\sin(700\pi t)}{\pi t}$$

$$= \frac{1}{2}\left[\frac{2\sin(500\pi t)\,.\,\sin(700\pi t)}{\pi^2 t^2}\right]$$

$$= \frac{1}{2}[\cos(700\pi t - 500\pi t) - \cos(700\pi t + 500\pi t)]$$

$$= \frac{1}{2\pi^2 t^2}[\cos(200\pi t) - \cos(1200\pi t)]$$

Maximum frequency component

$$f_m = \frac{1200\pi}{2\pi} = 600Hz$$

Nyquist sampling rate

$$f_{s_{min}} = 2f_m = 1200Hz$$

Option (c)

13. Consider a sample signal

$$y(t) = 5 \times 10^{-6} x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_S)$$

Where

$x(t) = 10\cos(8\pi \times 10^3)t \; and \; T_S = 100 \; \mu \; sec$
When y(t) is passed through an ideal low-pass filter with a cutoff frequency of 5 kHz, the output of the filter is
(a) $5 \times 10^{-6} \cos(8\pi \times 10^3)t$
(b) $5 \times 10^{-5} \cos(8\pi \times 10^3)t$
(c) $5 \times 10^{-1} \cos(8\pi \times 10^3)t$
(d) $100 \cos(8\pi \times 10^3)t$

[GATE 2002: 1 Mark]

Soln. Given, the sample signal

$$\boldsymbol{y(t) = 5 \times 10^{-6} x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_S)}$$

$$\boldsymbol{x(t) = 10\cos(8\pi \times 10^3)\, t}$$

$$\boldsymbol{T_S = 100\mu\mathrm{sec}}$$

Note, Fourier series expansion of impulse train is

$$\boldsymbol{\delta_{TS}(t) = \frac{1}{T_S}[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + - - -]}$$

Sampled signal can be written as

$$\boldsymbol{5 \times 10^{-6}.\, x(t). \frac{1}{T_S}[1 + 2\cos\omega_s\, t + - - -]}$$

$$= \frac{5 \times 10^{-6}}{100 \times 10^{-6}} . 10 \cos(8\pi \times 10^3)t \, [1 + 2 \cos \omega_s t + - -]$$

$$= 5 \times 10^{-1} \cos(8\pi \times 10^3) \, t[1 + 2 \cos \omega_s t + - - -]$$

It is passed through 5 KHz filter then the output will be

$$5 \times 10^3 \cos(8\pi \times 10^3) \, t$$

Option (c)

14. If E_b, the energy per bit of a binary digital signal, is 10^{-5} watt-sec and the one-sided power spectral density of the white noise, $N_0 = 10^{-6}$ W/Hz, then the output SNR of the matched filter is

(a) 26 dB
(b) 10 dB
(c) 20 dB
(d) 13 dB

[GATE 2003: 2 Marks]

Soln. Given

$$E_b = 10^{-5} \, watt \; sec$$

$$N_0 = 10^{-6} \, W/Hz$$

SNR of matched filter

$$= \frac{E_b}{N_0/2} = \frac{10^{-5}}{10^{-6}/2} = \frac{2 \times 10^{-5}}{10^{-6}} = 20$$

$$(SNR)_{dB} = 10 \log 20 = 13 dB$$

Option (d)

15. The transfer function of a zero-order hold is

(a) $\frac{1-exp(-T_S)}{S}$

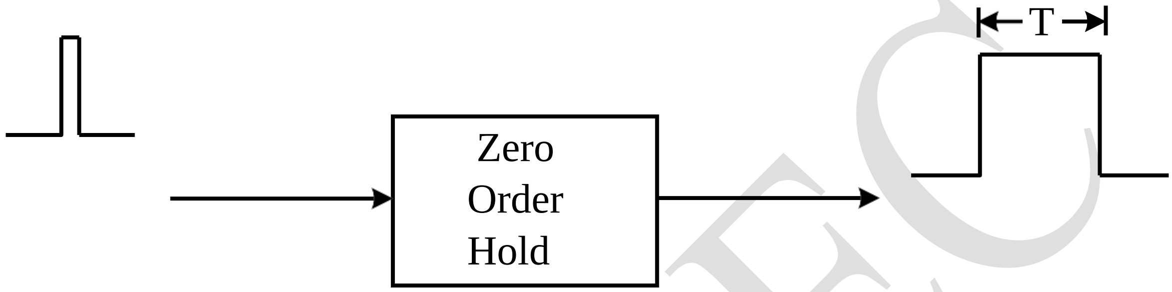
(b) $\frac{1}{S}$

(c) 1

(d) $\frac{1-exp(-T_S)}{S}$

[GATE 1988: 2 Marks]

Soln. Zero order holding circuit holds the input signal value for a period of T, for an input of short duration $\delta(t)$ it produces an output pulse duration T.



Input $x(t) = \delta(t) \quad so \quad X(s) = 1$

Output $y(t) = u(t) - u(t - T)$

$$So, y(s) = \frac{1}{s} - \frac{e^{-Ts}}{s} = \frac{1 - e^{-Ts}}{s}$$

$$H(s) = \frac{y(s)}{X(s)} = \frac{1 - e^{-Ts}}{s}$$

Option (a)

(b) Pulse Code Modulation (PCM)

16. An analog voltage in the range 0 to 8 V is divided in 16 equal intervals for conversion to 4-bit digital output. The maximum quantization error (in V) is ______________

[GATE 2014: 1 Mark]

Soln. Given,

Dynamic range or voltage range = 0 to 8 V

Number of levels = 16

Maximum quantization error

$$Q_e = \frac{step\ size\ (\Delta)}{2}$$

Where

$$\Delta = \frac{Dynamic\ range}{L}$$

$$= \frac{8}{2^4} = \frac{8}{16} = 0.5$$

$$Q_e = \frac{0.5}{2} = 0.25\ V$$

Quantization error is 0.25 V

17. Compression in PCM refers to relative compression of
   (a) higher signal amplitudes
   (b) lower signal amplitudes
   (c) lower signal frequencies
   (d) higher signal frequencies

[GATE 1999: 1 Mark]

Soln. In PCM, Companding results in making SNR uniform irrespective of signal amplitude level.

In formed from two words 'compressing' and 'expanding'

In PCM, analog signal values are rounded on a non-linear scale. The data is compressed before it is sent and then expanded at the receiving end using same non-linear scale.

So right option is compression of higher signal amplitudes

Option (a)

18. The line code that has zero dc component for pulse transmission of random binary data is

(a) non-return to zero (NRZ)
(b) return to zero (RN)
(c) alternate mark inversion (AM)
(d) none of the above

[GATE 1997: 1 Mark]

Soln. There are two types coding

- **Source coding techniques are used in PCM and DM. In this analog signal is converted to digital i.e. train of binary digits**
- **Line coding converts stream of binary digits into a formal or code which is more suitable for transmission over a cable or any other medium**

Alternate mark inversion (AMI) code has zero dc component for pulse transmission of random binary data

Option (c)

19. The signal to quantization noise ratio in an n-bit PCM system
(a) depends upon the sampling frequency employed
(b) is independent of the value of ‘n’
(c) increasing with increasing value of ‘n’
(d) decreases with the increasing value of ‘n’

[GATE 1995: 1 Mark]

Soln. Signal to quantization noise in an n bit PCM is

$$\left(\frac{S}{N_q}\right) = (SQNR) = \frac{3}{2}.2^{2n} - - - - - (1A)$$

Where n is number of bits in the word of binary PCM.

It can be written in dBs

$$(SQNR)_{dB} = \left(\frac{S}{N_q}\right)_{dB} = 1.76 + 6n - - - - (1B)$$

From above equations (S/N) increases with n

Option (c)

20. If the number bits per sample in a PCM system is increased from and n to (n+1), the improvement is signal to quantization nose ratio will be

(a) 3 dB (c) 2 n dB
(b) 6 dB (d) n dB

[GATE 1995: 1 Mark]

Soln. Note $\left(\frac{S}{N_q}\right)_{dB} = (1.76 + 6n)_{dB}$

$(SQNR)_1 = 1.76 + 6n$

$(SQNR)_2 = 1.76 + 6n(n+1) = 1.76 + 6n + 6$

$(SQNR)_2 - (SQNR)_1 = 1.76 + 6n + 6 - 1.76 - 6n = 6dB$

So for every one bit increase in bits per sample will result is 6 dB improvement in signal to quantization ratio

Option (b)

21. The bandwidth required for the transmission of a PCM signal increases by a factor of_____________when the number of quantization levels is increased from 4 to 64.

[GATE 1994: 1 Mark]

Soln. $(Bandwidth)_{PCM} = nf_m$

Where n – number of bits in PCM code

f_m – signal bandwidth

$n = log_2 L$

$n_1 = log_2 4 = 2$

$n_2 = log_2 64 = 6$

$(BW)_1 = n_1 f_m =$

$2f_m (BW)_2 = n_2 f_m =$

$6f_m$

$$\frac{(BW)_2}{(BW)_1} = \frac{6f_m}{2f_m} = 3\ times$$

$(BW)_2 = 3(BW)_1$

So, increase is 3 times

22. The number of bits in a binary PCM system is increased from n to n+1. As a result, the signal quantization noise ratio will improve by a factor

(a) $(n+1)/n$

(b) $2^{(n+1)/n}$

(c) $2^{2(n+1)/n}$

(d) Which is independent of n

[GATE 1996: 2 Marks]

Soln. $\boldsymbol{SQNR = \frac{3}{2}2^{2n}}$

For $\boldsymbol{n_1 = n}$

$\boldsymbol{n_2 = n + 1}$

$$\boldsymbol{(SQNR)_1 = \frac{3}{2}2^{2n}}$$

$$\boldsymbol{(SQNR)_2 = \frac{3}{2}2^{2(n+1)} = \frac{3}{2}2^{2n+2} = \frac{3}{2}[2^{2n}.\,2^2]}$$

$$\boldsymbol{\frac{(SQNR)_2}{(SQNR)_1} = \frac{2^2}{1} = 4}$$

So, $\boldsymbol{(SQNR)_2 = 4(SQNR)_1}$

Note that signal to quantization noise increases by factor of 4. So this is improvement in SQNR is independent of n

Option (d)

23. In a PCM system with uniform quantization, increasing the number of bits from 8 to 9 will reduce the quantization noise power by a factor of

(a) 9	(c) 4
(b) 8	(d) 2

[GATE 1998: 1 Mark]

Soln. Quantization noise in PCM is given by

$$N_q = \frac{\Delta^2}{12}$$

$$stepsize\ (\Delta) = \frac{voltage\ range}{2^n} = \frac{V_{p-p}}{2^n}$$

$$N_q = \frac{V_{p-b}^2}{12 \times 2^{2n}} \quad so,\ N_q \propto \frac{1}{2^{2n}}$$

$$\frac{(N_q)_2}{(N_q)_1} = \frac{2^{2n_1}}{2^{2n_2}} = \frac{2^{2\times 8}}{2^{2\times 9}} = \frac{2^{16}}{2^{18}} = \frac{1}{2^2} = \frac{1}{4}$$

$$or \quad (N_q)_2 = \frac{(N_q)_1}{4}$$

Quantization noise reduces by a factor of 4

Option (c)

24. The peak to peak input to an 8 bit PCM coder is 2 volts. The signal power to quantization noise power ratio (in dB) for an input of $0.5\cos\omega_m t$ is

(a) 47.8 (c) 95.6

(b) 43.8 (d) 99.6

[GATE 1999: 2 Marks]

Soln. Given

$$V_{p-p} = 2\ volts$$

No. of bits = 8

$$n = log_2 L$$

Or $8 = log_2 L\ or\ L = 2^7 = 128\ levels$

$$\left(\frac{S}{N_q}\right)_{dB} = 1.76 + 6n$$

$$= \mathbf{1.76 + 6 \times 7 = 43.8}\ \boldsymbol{dB}$$

Option (b)

25. A signal is sampled at 8 KHz and is quantized using 8-bit uniform quantizer. Assuming SNR_q for a sinusoidal signal, the correct statement for PCM signal with a bit rate of R is
(a) $R = 32\ kbps,\ SNR_q = 25.8\ dB$
(b) $R = 64\ kbps,\ SNR_q = 49.8\ dB$
(c) $R = 64\ kbps,\ SNR_q = 55.8\ dB$
(d) $R = 32\ kbps,\ SNR_q = 49.8\ dB$

[GATE 2003: 2 Marks]

Soln. **Given sampling rate = 8 KHz**

Then bit rate $= \boldsymbol{nf_s = 8 \times 8kHz = 64\ \ kbps}$

$$\boldsymbol{SNR_q = (1.76 + 6.02\ .\ n)dB}$$

$$= \mathbf{1.76 + 6.02 \times 8}$$

$$= \mathbf{49.8}\ \boldsymbol{dB}$$

Option (b)

26. In a PCM system, if the code word length is increased from 6 to 8 bits, the signal to quantization noise ratio improves by the factor
(a) 8/6
(b) 12
(c) 16
(d) 8

[GATE 2004: 1 Mark]

Soln. Note that

$$\boldsymbol{\left(\frac{S}{N_q}\right)_{n=6} = 2^{2\times 6} = 2^{12}}$$

$$\boldsymbol{\left(\frac{S}{N_q}\right)_{n=8} = 2^{2\times 8} = 2^{16}}$$

$$\frac{\left(\frac{S}{N_q}\right)_{n=8}}{\left(\frac{S}{N_q}\right)_{n=6}} = \frac{2^{16}}{2^{12}} = 2^4 = 16$$

Option (c)

27. A sinusoidal signal with peak to peak amplitude of 1.536 V is quantized into 128 levels using a midrise uniform quantizer. The quantization noise power is

(a) 0.768 V
(b) $48 \times 10^{-6} V^2$
(c) $12 \times 16^{-6} V^2$
(d) 3.072 V

[GATE 2003: 2 Marks]

Soln. Note

$$\boldsymbol{stepsize}(\Delta) = \frac{\boldsymbol{V_{p-p}}}{\boldsymbol{No.\,of\ levels}} = \frac{\mathbf{1.536}}{\mathbf{128}} = \mathbf{0.012}\boldsymbol{V}$$

Quantization noise

$$(\boldsymbol{N_q}) = \frac{\Delta^2}{\mathbf{12}} = \frac{(\mathbf{0.012})^2}{\mathbf{12}} = \mathbf{12} \times \mathbf{10}^{-6}\boldsymbol{V}^2$$

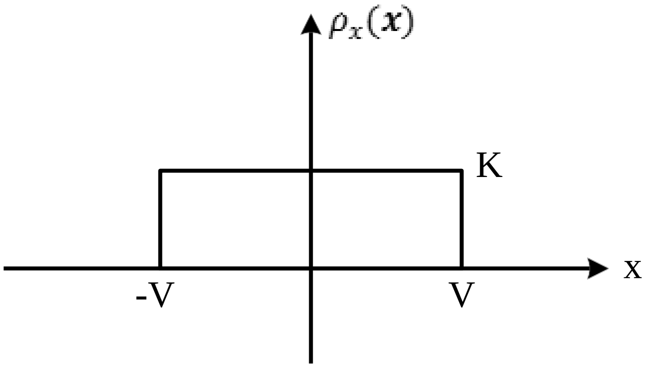
Option (c)

28. A signal having uniformly distributed amplitude in the interval (-V to +V), is to be encoded using PCM with uniform quantization. The signal to quantizing noise ratio is determined by the

(a) dynamic
(b) sampling
(c) number of quantizing levels
(d) power spectrum of signal

[GATE 1988: 2 Marks]

Soln. The signal is uniformly distributed in the interval –V to +V, the PDF is shown in figure.



Area under PDF is unity

$$K[V-(-V)] = 1 \quad or \quad 2VK = 1 \quad or\ K = \frac{1}{2V}$$

So $\rho_x(x) = \begin{cases} \frac{1}{2V} & -V\ to\ V \\ 0\ otherwise \end{cases}$

Signal power

$$= s = \int_{-\infty}^{\infty} x^2\, \rho_x(x)dx = \int_{-V}^{V} x^2 . \frac{1}{2V} . dx = \frac{1}{2V}\left[\frac{x^3}{3}\right]_{-V}^{V} = \frac{1}{2V}\left[\frac{V^3}{3} - \frac{(-V^3)}{3}\right]$$

$$S = \frac{1}{2V} . \frac{2V^3}{3} = \frac{V^2}{3}$$

Quantization noise $(N_q) = \frac{\Delta^2}{12}$

Where

$$\Delta = \frac{V_{p-p}}{L} = \frac{V_{p-p}}{2^n}$$

$$N_q = \left(\frac{2V}{2^n}\right)^2 . \frac{1}{12} = \frac{4V^2}{12 \times 2^{2n}} = \frac{V^2}{3 \times 2^n}$$

$$\left(\frac{S}{N}\right)_q = \frac{V^2/3}{V^2/3 \times 2^n} = 2^n$$

So,

$$\left(\frac{S}{N}\right)_q \propto 2^{2n}$$

So, signal to quantizing noise ratio is determined by number of quantizing levels

Option (c)

29. In a PCM systems, the signal $m(t) = \{\sin(100\,\pi t + \cos(100\,\pi t))\}$ is sampled at the Nyquist rate. The samples are processed by a uniform quantizer with step size 0.75V. The minimum data rate of the PCM system in bits per seccond is

[GATE 2014: 2 Marks]

Soln. Given, $\boldsymbol{m(t) = \sin 100\pi t + \cos 100\pi t = \sqrt{2}.\cos[100\pi t + \emptyset]}$

Step size $\Delta = \boldsymbol{0.75V}$

$$\Delta = \frac{V_{p-p}}{L} = \frac{\sqrt{2} - (-\sqrt{2})}{L} = \frac{2\sqrt{2}}{L}$$

Or,

$$L = \frac{2\sqrt{2}}{0.75} \cong 4 \quad so, \quad n = 2$$

Frequency of signal f = 50 Hz

Nyquist rate = 100

Bit rate $\boldsymbol{(R_b) = 2fs = 2 \times 100}$

$$= 200\ bit/sec$$

$$= 200\ bps$$

30. An analog signal is band-limited to 4 KHz, sampled at the Nyquist rate and the samples levels are assumed to be independent and equally probable. If we transmit two quantized samples per second, the information rate is

(a) 1 bit/sec
(b) 2 bits/sec
(c) 3 bits/sec
(d) 4 bits/sec

[GATE 2011: 1 Mark]

Soln. Signal is band limited to 4 KHz

Nyquist rate $= 2 \times f_m = 8KHz$

Levels = 4 i.e. $2^n = L \ or \ n = 2$

The number of bits = 2

Each sample requires 2 bits.

Two samples per second are transmitted so, the number of bits per second $2 \times 2 = 4 \ bits/sec$

Option (d)

31. In a baseband communication link, frequencies up to 3500 Hz are used for, signaling. Using a raised cosine pulse with 75% excess bandwidth and for no inter-symbol interference, the maximum possible signaling rate in symbols per second is

(a) 1750
(b) 2625
(c) 4000
(d) 5250

[GATE 2012: 1 Mark]

Soln. Given

Signaling frequency (f) = 3500 Hz

Excess bandwidth used is $B = 0.75 \times 3500 = 2625 \ Hz$

We know that

$$\text{Bandwidth} \geq \frac{R_b}{2}$$

Where R_b **is the data rate**

$$\textbf{Minimum bandwidth required}(B) = \frac{R_b}{2}$$

$$or, \quad B = \frac{R_b}{2}$$

$$or, R_b = 2B = 2 \times 2625 = 5250 \ Hz$$

Option (d)

(c) Delta Modulation

32. In delta modulation, the slope overload distortion can be reduced by
    (a) decreasing the step size
    (b) decreasing the granular noise
    (c) decreasing the sampling noise
    (d) increasing the step size

[GATE 2007: 2 Marks]

Soln. When the slope of analog signal is much higher than that of approximated digital signal, then this difference is called slope overload distortion.

Condition to avoid slop overload in delta modulation is

$$\frac{\Delta}{Ts} \geq \frac{d}{dt}.m(t)$$

Where, $\Delta - \boldsymbol{Step\ size}$

$\boldsymbol{Ts - sampling\ interval}$

$\boldsymbol{m(t) - signal}$

From above equation we observe that if step size is increased, slope overload distortion can be avoided.

33. In the output of a DM speech encoder, the consecutive pulses are of opposite polarity during time interval $t_1 \leq t \leq t_2$. This indicates that during this interval
    (a) the input to the modulator is essentially constant
    (b) the modulator is going through slope overload
    (c) the accumulator is in saturation
    (d) the speech signal is being sampled at the Nyquist rate

[GATE 2004: 1 Mark]

Soln. Given

During the interval $\boldsymbol{t_1 \leq t \leq t_2}$

The consecutive pulses of encoder are of opposite polarity.

In between the two adjacent sample values, if the baseband signal changes by an amount less than the step size, the output of Delta Modulator is sequence of alternate positive and negative pulses.

This small change in base band signal indicates that the baseband is almost constant

Option (a)

34. The input to a linear delta modulator having a step-size $\Delta = 0.628$ is a sine wave with frequency f_m and peak amplitude E_m. If the sampling frequency $f_s = 40$ KHz, the combination of the sine-wave frequency and the peak amplitude, where slope overload will take place is

	E_m	f_m
(a)	0.3 V	8 KHz
(b)	1.5 V	4 KHz
(c)	1.5 V	3 KHz
(d)	3.0 V	1 KHz

[GATE 2003: 2 Marks]

Soln. Given

$$\Delta = \mathbf{0.628}$$

$$\boldsymbol{m(t) = E_m \sin 2\pi f_m t}$$

Slope overload takes place

When,

$$\frac{\Delta}{\boldsymbol{Ts}} \le \frac{\boldsymbol{d}}{\boldsymbol{dt}}.\boldsymbol{m(t)}$$

$$\boldsymbol{or,} \qquad \frac{\Delta}{\boldsymbol{Ts}} \le \boldsymbol{2\pi f_m . E_m}$$

$$\boldsymbol{or,} \qquad \Delta . \boldsymbol{f_s} \le \boldsymbol{2\pi f_m E_m}$$

$$\boldsymbol{or,} \qquad \Delta . \mathbf{40 \times 10^3 \le 2\pi} \boldsymbol{f_m}$$

$$\boldsymbol{or,} \quad \mathbf{0.628 \times 40 \times 10^3 \le 6.28} \boldsymbol{f_m E_m}$$

$$or, 4 \times 10^3 \leq f_m E_m$$

(a) $\mathbf{0.3 \times 8KHz = 2.4KHz}$
(b) $\mathbf{1.5 \times 4KHz = 6KHz}$
(c) $\mathbf{1.5 \times 2KHz = 3KHz}$
(d) $\mathbf{3 \times 1KHz = 3KHz}$

Option (b)

35. The minimum step-size required for a Delta-Modulation operating at 32K samples/sec to track the signal (here u(t) is the unit-step function) $x(t) = 125t\{u(t) - u(t-1)\} + (250 - 125t)\{u(t-1) - u(t-2)\}$ so that slope-overload is avoided would be

(a) 2^{-10}
(b) 2^{-8}
(c) 2^{-6}
(d) 2^{-4}

[GATE 2006: 2 Marks]

Soln.

Given

$$\boldsymbol{m(t) = 125t[u(t) - u(t-1)] + (250 - 125t)[u(t-1) - u(t-2)]}$$

To avoid slope overload

$$\frac{\Delta}{Ts} \geq \frac{d}{dt} m(t)$$

$$\Delta \times 32 \times 1024 \geq 125$$

$$or, \Delta . 2^{15} \geq 125$$

$$or, \quad \Delta \geq \frac{2^7}{2^{15}}$$

$$or, \Delta \geq 2^{-8}$$

Option (b)

Angle Modulated Systems

1. Consider an FM wave
   $f(t) = \cos[2\pi f_c t + \beta_1 \sin 2\pi f_1 t + \beta_2 2\pi f_2 t]$
   The maximum deviation of the instantaneous frequency from the carrier frequency f_c is

   (a) $\beta_1 f_1 + \beta_2 f_2$ (c) $\beta_1 + \beta_2$
   (b) $\beta_1 f_2 + \beta_2 f_1$ (d) $f_1 + f_2$

[GATE 2014: 1 Mark]

Soln. The instantaneous value of the angular frequency

$$\boldsymbol{\omega_i = \omega_c + \frac{d}{dt}(\beta_1 \sin 2\pi f_1 t + \beta_2 \sin 2\pi f_2 t)}$$

$$\boldsymbol{\omega_c + 2\pi\beta_1 f_1 \cos 2\pi f_1 t + 2\pi\beta_2 f_2 \cos 2\pi f_2 t}$$

$$\boldsymbol{f_i = f_c + \beta_1 f_1 \cos 2\pi f_1 t + \beta_2 f_2 \cos 2\pi f_2 t}$$

Frequency deviation $(\Delta_f)_{max} = \boldsymbol{\beta_1 f_1 + \beta_2 f_2}$

Option (a)

2. A modulation signal is $y(t) = m(t)\cos(40000\pi t)$, where the baseband signal m(t) has frequency components less than 5 kHz only. The minimum required rate (in kHz) at which y(t) should be sampled to recover m(t) is ___

[GATE 2014: 1 Mark]

Soln. **The minimum sampling rate is twice the maximum frequency called Nyquist rate**

The minimum sampling rate (Nyquist rate) = 10K samples/sec

3. Consider an angle modulation signal $x(t) = 6cos[2\pi \times 10^3 + 2\sin(8000\pi t) + 4\cos(8000\pi t)]V$. The average power of $x(t)$ is

(a) 10 W (c) 20 W

(b) 18 W (d) 28 W

[GATE 2010: 1 Mark]

Soln. The average power of an angle modulated signal is

$$\frac{A_c^2}{2} = \frac{6^2}{2}$$

=18 W

Option (b)

4. A modulation signal is given by $s(t) = e^{-at}\cos[(\omega_c + \Delta_\omega)t]\, u(t)$, where, ω_c and $\Delta\omega$ are positive constants, and $\omega_c \gg \Delta\omega$. The complex envelope of s(t) is given by

(a) $\exp(-at)\ exp[j(\omega_c + \Delta\omega)]u(t)$

(b) $\exp(-at)\ \exp(j\Delta\omega t)\ u(t)$

(c) $exp(j\Delta\omega t)u(t)$

(d) $exp[(j\omega_c + \Delta\omega)t]$

[GATE 1999: 1 Mark]

Soln. $s(t) = e^{-at}\cos[(\omega_c + \Delta_\omega)t]u(t)$

Complex envelope $\tilde{s}(t) = s(t)e^{-j\omega ct}$

$$= [e^{-at}e^{j(\omega_c+\Delta_\omega)t}.\, u(t)]e^{-j\omega_c t}$$

$$= e^{-at}e^{j\Delta\omega t}u(t)$$

Option (b)

5. A 10 MHz carrier is frequency modulated by a sinusoidal signal of 500 Hz, the maximum frequency deviation being 50 KHz. The bandwidth required. as given by the Carson's rule is ________

[GATE 1994: 1 Mark]

Soln. By carson's rule

$$\boldsymbol{BW = 2(\Delta_f + f_m)}$$

$$\boldsymbol{= 2(50 + 0.5)}$$

$$\boldsymbol{= 101\ KHz}$$

6. $v(t) = 5[\cos(10^6\pi t) - \sin(10^3\pi t) \times \sin(10^6\pi t)]$ represents
   (a) DSB suppressed carrier signal
   (b) AM signal
   (c) SSB upper sideband signal
   (d) Narrow band FM signal

[GATE 1994: 1 Mark]

Soln. $\boldsymbol{v(t) = 5\cos(10^6\pi t) - \frac{5}{2}\cos(10^6 - 10^3)\pi t + \frac{5}{2}\cos(10^6 + 10^3)\pi t}$

Carrier and upper side band are in phase and lower side band is out of phase with carrier

The given signal is narrow band FM signal

Option (d)

7. The input to a coherent detector is DSB-SC signal plus noise. The noise at the detector output is
   (a) the in-phase component
   (b) the quadrature-component
   (c) zero
   (d) the envelope

[GATE 2003: 1 Mark]

Soln. The coherent detector rejects the quadrature component of noise therefore noise at the output has in phase component only.

Option (a)

8. An AM signal and a narrow-band FM signal with identical carriers, modulating signals and modulation indices of 0.1 are added together. The resultant signal can be closely approximated by

(a) Broadband FM
(b) SSB with carrier
(c) DSB-SC
(d) SSB without carrier

[GATE 2004: 1 Mark]

Soln. $$\boldsymbol{V_{AM}(t) = A\cos\omega_c t + \frac{0.1A}{2}\cos(\omega_c+\omega_m)t + \frac{0.1A}{2}\cos(\omega_c-\omega_m)t}$$

$$\boldsymbol{V_{FM}(t)(narrowband)}$$

$$\boldsymbol{= A\cos\omega_c t + \frac{0.1A}{2}\cos(\omega_c+\omega_m)t - \frac{0.1A}{2}\cos(\omega_c-\omega_m)t}$$

$$\boldsymbol{V_{AM}(t) + V_{FM}(t) = 2A\cos\omega_c t + 0.1A\cos(\omega_c+\omega_m)t}$$

The resulting signal is SSB with carrier

Option (b)

9. The List-I (lists the attributes) and the List-II (lists of the modulation systems). Match the attribute to the modulation system that best meets it.

List-I

(A) Power efficient transmission of signals
(B) Most bandwidth efficient transmission of voice signals
(C) Simplest receiver structure
(D) Bandwidth efficient transmission of signals with significant dc component

List-II

(1) Conventional AM

(2) FM

(3) VSB

(4) SSB-SC

	A	B	C	D
(a)	4	2	1	3
(b)	2	4	1	3
(c)	3	2	1	4
(d)	2	4	3	1

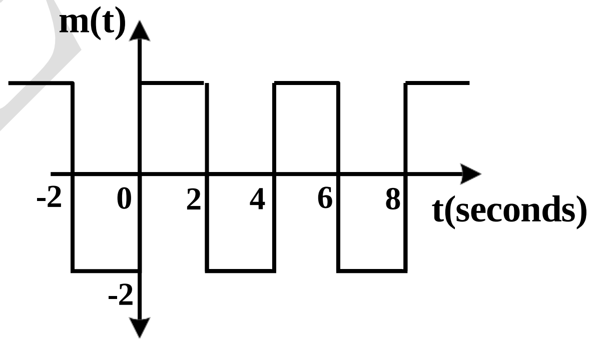
[GATE 2011: 1 Mark]

Soln. FM is the most power efficient transmission of signals AM has the simplest receiver. Vestigial sideband is bandwidth efficient transmission of signals with sufficient dc components. Single sideband, suppressed carrier (SSB-SC) is the most bandwidth efficient transmission of voice signals.

Option (b)

10. The signal m(t) as shown I applied both to a phase modulator (with k_p as the phase constant) and a frequency modulator with (k_f as the frequency constant) having the same carrier frequency

The ratio k_p/k_f (in rad/Hz) for the same maximum phase deviation is



(a) 8π (c) 2π

(b) 4π (d) π

[GATE 2012: 2 Marks]

Soln. For a phase modulator, the instantaneous value of the phase angle ψ_i is equal to phase of an unmodulated carrier $\omega_c(t)$ plus a time varying component proportional to modulation signal m(t)

$$\psi_{PM}(t) = 2\pi f_c t + k_p m(t)$$

Maximum phase deviation $(\psi_{PM})_{max}$

$$= K_p \max m(t) = 2K_p$$

For a frequency modulator, the instantaneous value of the angular frequency

$$\omega_i = \omega_c + 2\pi K_f m(t)$$

The total phase of the FM wave is

$$\psi_{FM} = \int \omega_i dt$$

$$= \omega_c t + 2\pi K_f \int_0^t m(t)dt$$

$$(\psi_{FM})_{max} = 2\pi K_f \int_0^2 2dt$$

$$= 8\pi K_f$$

$$\frac{K_P}{K_f} = \frac{8\pi}{2}$$

$$= \boldsymbol{4\pi}$$

Option (b)

11.Consider the frequency modulated signal

$$10[\cos 2\pi \times 10^5 t + 5\sin(2\pi \times 1500t) + 7.5\sin(2\pi \times 1000t)]$$

with carrier frequency of 10^5 Hz. The modulation index is

(a) 12.5 (c) 7.5

(b) 10 (d) 5

[GATE 2008: 2 Marks]

Soln. Frequency modulated signal

$$\mathbf{10\cos[2\pi \times 10^5 t + 5\sin(2\pi \times 1500t) + 7.5\sin(2\pi \times 10000t)]}$$

The instantaneous value of the angular frequency

$$\boldsymbol{\omega_i = \omega_c + \frac{d}{dt}[5\sin(2\pi \times 1500t) + 7.5\sin(2\pi \times 1000t)]}$$

Frequency deviation

$$\boldsymbol{\Delta_\omega = 5 \times 2\pi \times 1500\cos(2\pi \times 1500) + 7.5 \times 2\pi \times 1000\cos(2\pi \times 1000t)}$$

$$\boldsymbol{(\Delta_\omega)_{max} = 2\pi(7500 + 7500)}$$

Frequency deviation $\boldsymbol{(\delta) = \frac{(\Delta_\omega)_{max}}{2\pi} = 15000Hz}$

Modulation index $\boldsymbol{m_f = \frac{15000}{1500}}$

= 10

Option (b)

12. A message signal with bandwidth 10 KHz is Lower-Side Band SSB modulated with carrier frequency $f_{c_1} = 10^6 Hz$. The resulting signal is then passed through a narrow-band frequency Modulator with carrier frequency $f_{c2} = 10^9 Hz$.
The bandwidth of the output would be

(a) $4 \times 10^4 Hz$
(b) $2 \times 10^6 Hz$
(c) $2 \times 10^9 Hz$
(d) $2 \times 10^{10} Hz$

[GATE 2006: 2 Marks]

Soln. Lower side band frequency $= \mathbf{10^3 - 10}$

$$= \mathbf{990\ KHz}$$

Considering this as the baseband signal, the bandwidth of narrow band FM

$$= \mathbf{2 \times 990\ KHz}$$

$$\approx \mathbf{2MHz}$$

Option (b)

13. A device with input $x(t)$ and output $y(t)$ is characterized by: $y(t) = x^2(t)$. An FM signal with frequency deviation of 90 KHz and modulating signal bandwidth of 5 KHz is applied to this device. The bandwidth of the output signal is

(a) 370 KHz
(b) 190 KHz
(c) 380 KHz
(d) 95 KHz

[GATE 2005: 2 Marks]

Soln. Frequency deviation $\Delta_f = \mathbf{90KHz}$

Modulating signal bandwidth = 5 KMz

When FM signal is applied to doubler frequency deviation doubles.

$$\boldsymbol{B.W = 2(\Delta_f + f_m)}$$

$$= \mathbf{2(180 + 5)}$$

$$= \mathbf{370\ \mathit{KHz}}$$

Option (a)

14. An angle-modulation signal is given by
$s(t) = \cos(2\pi \times 2 \times 10^6 t + 2\pi \times 30 \sin 150t + 2\pi \times 40 \cos 150t)$
The maximum frequency and phase deviations of s(t) are
(a) 10.5KHz, 140π rad
(b) 6 KHz, 80π rad
(c) 10.5 KHz, 100π rad
(d) 7.5 KHz, 100π rad

[GATE 2002: 2 Marks]

Soln. The total phase angle of the carrier

$$\boldsymbol{\psi = \omega_c t + \theta_0}$$

$$\boldsymbol{\psi = 2\pi \times 2 \times 10^6 + 2\pi \times 30 \sin 150\ t + 2\pi \times 40 \cos 150t}$$

Instantaneous value of angular frequency $\boldsymbol{\omega_i}$

$$\boldsymbol{\omega_i = \omega_c + \frac{d}{dt}(2\pi \times 30 \sin 150t + 2\pi \times 40 \cos 150t)}$$

$$\boldsymbol{= \omega_c + 2\pi \times 30 \times 150 \cos 150t - 2\pi \times 40 \times 150 \sin 150t}$$

$$\boldsymbol{= \omega_c + 2\pi \times 4500 \cos 150t - 2\pi \times 6000 \sin 150\ t}$$

Frequency deviation $\boldsymbol{\Delta_\omega = 2\pi \times 1500[3 \cos 150t - 4 \sin 150t]}$

$$\boldsymbol{= 3000\pi\sqrt{3^2 + 4^2}\, rad/sec}$$

$$\boldsymbol{= 1500\pi\, rad/sec}$$

$$\boldsymbol{= 2\pi \times 7.5K\, rad/sec}$$

$$\boldsymbol{\Delta_f = \frac{\Delta_\omega}{2\pi} = 7.5KHz}$$

Phase deviation $\boldsymbol{\Delta_\psi}$ is proportional to $\boldsymbol{\theta_0}$

$$\Delta\psi = 2\pi\sqrt{30^2 + 40^2}$$

$$= 2\pi \times 50 = 100\pi\, rad$$

Option (d)

15. In a FM system, a carrier of 100 MHz is modulated by a sinusoidal signal of 5 KHz. The bandwidth by Carson's approximation is 1MHz. If y(t) = (modulated waveform)^3, then by using Carson's approximation, the bandwidth of y(t) around 300 MHz and the spacing of spectral components are, respectively.

(a) 3 MHz, 5 KHz
(b) 1 MHz, 15 KHz
(c) 3 MHz, 15 KHz
(d) 1 MHz, 5 KHz

[GATE 2000: 2 Marks]

Soln. In an FM signal, adjacent spectral components will get separated by modulating frequency $f_m = 5KHz$

$$BW = 2(\Delta_f + f_m) = 1MHz$$

$$\Delta_f + f_m = 500\ KHz$$

$$\Delta_f = 495\ KHz$$

The n^{th} order non-linearity makes the carrier frequency and frequency deviation increased by n-fold, with baseband frequency f_m unchanged.

$$(\Delta_f)new = 3 \times 495$$

$$= 1485\ KHz$$

$$New\ BW = 2(1485 + 5) \times 10^3$$

$$= 2.98\ MHz$$

$$\approx 3\ MHz$$

Option (a)

16. An FM signal with a modulation index 9 is applied to a frequency tripler. The modulation index in the output signal will be

(a) 0
(b) 3
(c) 9
(d) 27

[GATE 1996: 2 Marks]

Soln. The frequency modulation index β is multiplied by n in ntimes frequency multiplier.

$$So,\ \beta' = 3 \times 9$$

$$= 27$$

Option (d)

17. A signal $x(t) = 2\ \cos(\pi.\ 10^4 t)$ volts is applied to an FM modulator with the sensitivity constant of 10 KHz/volt. Then the modulation index of the FM wave is

(a) 4
(b) 2
(c) $4/\pi$
(d) $2/\pi$

[GATE 1989: 2 Marks]

Soln. Modulation index

$$\beta = \frac{K_f A_m}{f_m}$$

$$K_f = 10KHz/volt$$

A_m is the amplitude of modulating signal

f_m is the modulating frequency

$$\beta = \frac{10 \times 10^3 \times 2}{\frac{\pi \times 10^4}{2\pi}} = 4$$

Option (a)

18. A carrier $A_C \cos \omega_c t$ is frequency modulated by a signal $E_m \cos \omega_m t$.The modulation index is m_f. The expression for the resulting FM signal is

(a) $A_c \cos[\omega_c t + m_f \sin \omega_m t]$

(b) $A_c \cos[\omega_c t + m_f \cos \omega_m t]$

(c) $A_c \cos[\omega_c t + 2\pi m_f \sin \omega_m t]$

(d) $A_C \cos[\omega_c t + \frac{2\pi m_f E_m}{\omega_m} \cos \omega_m t]$

[GATE 1989: 2 Marks]

Soln. The frequency modulated signal

$$V_{FM}(t) = A_c \cos[\omega_c t + K_f \int m(t)dt]$$

K_f is the frequency sensitivity of the modulator

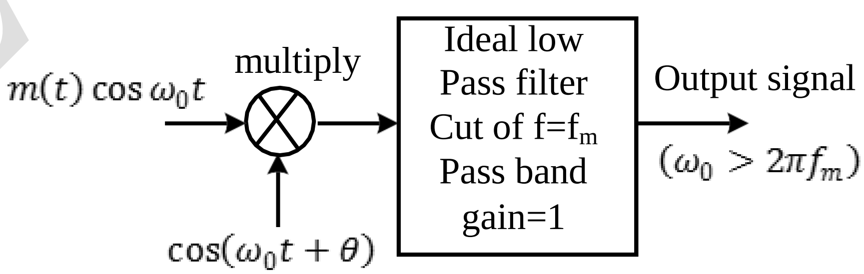
$$\int m(t)dt = \int E_m \cos \omega_m t\, dt = \frac{E_m \sin \omega_m t}{\omega_m}$$

$$V_{FM}(t) = A_c \cos\left[\omega_c t + \frac{K_f E_m}{\omega_m} \sin \omega_m t\right]$$

$= A_c \cos[\omega_c t + m_f \sin \omega_m t]$ **where m_f is the modulation index**

Option (a)

19. A message m(t) bandlimited to the frequency f_m has a power of P_m. The power of the output signal in the figure is



(a) $\frac{P_m \cos\theta}{2}$

(b) $\frac{Pm}{4}$

(c) $\frac{P_m sin^2\theta}{4}$

(d) $\frac{P_m cos^2\theta}{4}$

[GATE 2000: 2 Marks]

Soln. Output of the multiplier $= \boldsymbol{m(t)\cos\omega_0 t\, cos(\omega_0 t + \theta)}$

$$= \frac{\boldsymbol{m(t)}}{\boldsymbol{2}}[\boldsymbol{\cos(2\omega_0 t + \theta) + \cos\theta}]$$

Output of LPF $\boldsymbol{V_0(t) = \frac{m(t)}{2}\cos\theta}$

$$= \frac{1}{2}\cos\theta\, m(t)$$

Power of output signal

$$= \lim_{T\to\infty}\frac{1}{T}\int_0^T V_0^2(t)dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int \frac{Cos^2\theta}{4} m^2(t)dt$$

$$= \frac{cos^2\theta}{4}\lim_{T\to\infty}\frac{1}{T}\int_0^T m^2(t)dt$$

$$= \frac{cos^2\theta}{4} P_m$$

Option (d)

20. c(t) and m(t) are used to generate an FM signal. If the peak frequency deviation of the generated FM signal is three times the transmission bandwidth of the AM signal , then the coefficient of the term

$5\cos[2\pi(1008 \times 10^3 t)]$ in the FM signal (in terms of the Bessel coefficients) is

(a) $5J_4(3)$

(b) $\frac{5}{2}J_8(3)$

(c) $\frac{5}{2}J_8(4)$

(d) $5J_4(6)$

[GATE 2003: 2 Marks]

Soln.

$$\boldsymbol{V_{FM}(t) = \sum_{n=-\infty}^{\infty} AJ_n(m_f)\cos(\omega_c + n\omega_m)t}$$

Peak frequency deviation of FM signal is three times the bandwidth of AM signal

$$\boldsymbol{\delta_f = 3 \times 2f_m = 6f_m}$$

Modulation index

$$\boldsymbol{m_f = \frac{\delta_f}{f_m} = \frac{6f_m}{f_m} = 6}$$

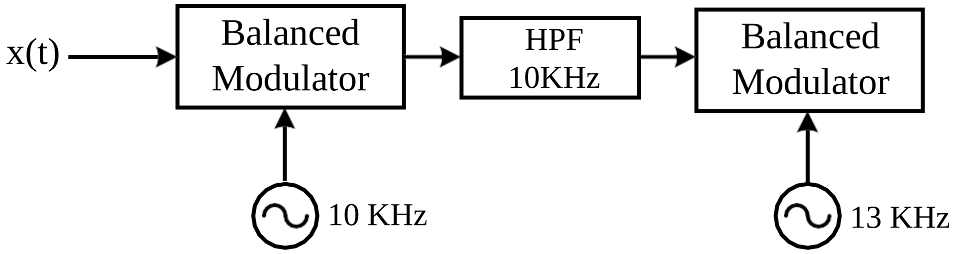
$$\boldsymbol{5\cos[2\pi(1008 \times 10^3 t)] = 5\cos[2\pi(1000 + 4 \times 2) \times 10^3]}$$

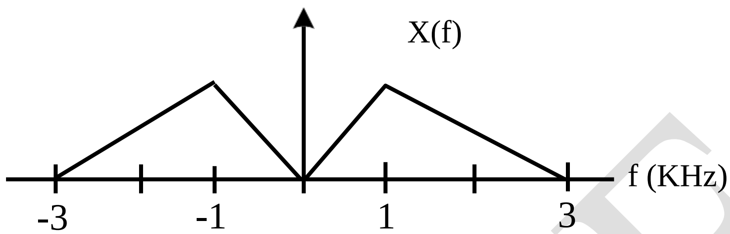
$$\boldsymbol{n = 4}$$

The required coefficient is $\boldsymbol{5J_4(6)}$

Option (d)

21. Consider a system shown in the figure. Let $X(f)$ and $Y(f)$ denote the Fourier transforms of $x(t) and$ $y(t)$ respectively. The ideal HPF has the cutoff frequency 10 KHz.





The positive frequencies where Y(f) has spectral peaks are

(a) 1 KHz and 24 KHz
(b) 2 KHz and 24 KHz
(c) 1 KHz and 14 KHz
(d) 2 KHz and 14 KHz

[GATE 2004: 2 Marks]

Soln. Input signal x(f) has the peaks at 1KHz and -1Mhz.

The output of balanced modulator will have peaks at

$$\boldsymbol{f_c \pm 1, f_c \pm (-1) f_c \pm 1 = 10 \pm 1}$$

$$\boldsymbol{= 11 \; and \; 9 \; KHz}$$

$$\boldsymbol{f_c \pm (-1) = 10 \pm (-1) = 9 \; KHz \; and \; 11 \; KHz}$$

9 MHz will be filtered out by the HPF

After passing through 13 KHz balanced modulator, the signal will have $\boldsymbol{13 \pm 11}$ frequencies.

$$\boldsymbol{y(f) = 24 \; K \; and \; 2 \; K}$$

Option (b)

Common Data for Questions 22 & 23

Consider the following Amplitude Modulated (AM) signal,

Where $f_m < B \quad X_{AM}(t) = 10(1 + 0.5 \sin 2\pi f_m t) \cos 2\pi f_c t$

22. The average side-band power for the AM signal given above is

(a) 25
(b) 12.5
(c) 6.25
(d) 3.125

[GATE 2006: 2 Marks]

Soln. The average sideband power for the AM signal is

$$P_{SB} = P_c \frac{ma^2}{2}$$

$$P_c \rightarrow carrier\ power$$

$$m_a \rightarrow modulation\ index$$

$$P_c = \frac{Ac^2}{2} = \frac{10^2}{2}$$

$$= 50\omega$$

$$m_a = 0.5$$

So,

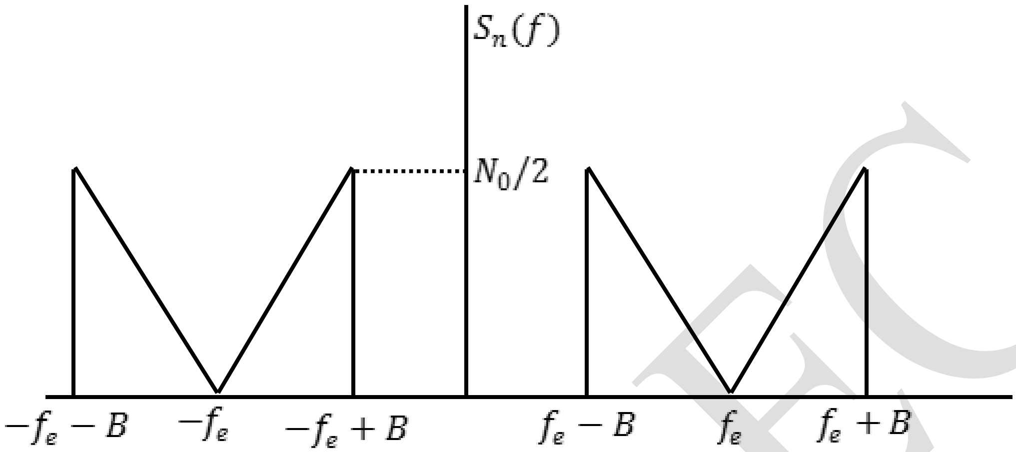
$$P_{SB} = 50 \frac{(0.5)^2}{2}$$

$$= \frac{50 \times 0.25}{2}$$

$$= 6.25\ watts$$

Option (c)

23. The AM signal gets added to a noise with Power Spectral Density $S_n(f)$ given in the figure below. The ratio of average sideband power to mean noise power would be:



(a) $\frac{25}{8N_0B}$

$\frac{25}{2N_0B}$

(b) $\frac{25}{4N_0B}$

(d) $\frac{25}{N_{0B}}$

(c)

[GATE 2006: Marks]

Soln. The AM signal gets added to a noise with spectral density $s_n(f)$

The noise power

$$P_T = \int_{-\infty}^{\infty} s_n(f)df$$

$$= 2\int_{0}^{\infty} s_n(f)df$$

Noise power $= 4\left[\frac{1}{2}\times\frac{B}{1}\times\frac{N_0}{2}\right]$

$$= N_0B$$

Power in sidebands P_{SB} $= \frac{25}{4}\ watts$

$$\frac{P_{SB}}{noise\ power} = \frac{25}{4N_0B} \qquad option\ (b)$$

Information Theory and Coding

1. The capacity of a band-limited additive white Gaussian (AWGN) channel is given by
$C = W log_2\ (1 + \frac{P}{\sigma^2 W})$ bits per second(bps), where W is the channel bandwidth, P is the average power received and σ^2 is the one-sided power spectral density of the AWGN.
For a fixed $\frac{P}{\sigma^2} = 1000$, the channel capacity (in kbps) with infinite bandwidth ($W \rightarrow \infty$) is approximately
(a) 1.44
(b) 1.08
(c) 0.72
(d) 0.36

[GATE 2014: 1 Mark]

Soln.

$$C = W\ log_2\ (1 + \frac{P}{\sigma^2 W})$$

$$= \frac{P}{\sigma^2} \times \frac{\sigma^2}{P} W\ log_2\ (1 + \frac{P}{\sigma^2 W})$$

$$= \frac{P}{\sigma^2} \times \frac{\sigma^2}{P} W\ log_2\ (1 + \frac{1}{\sigma^2 W/_P})$$

$$= \frac{P}{\sigma^2} \lim_{\rightarrow\infty} [x\ log_2\ (1 + \frac{1}{x})] \qquad where\ x = \frac{\sigma^2 W}{P}$$

$$= \frac{P}{\sigma^2}\ log_2 e$$

$$= 1.44 \times \frac{P}{\sigma^2}$$

$$= 1.44 \times 1000 = 1.44\ kbps$$

Option (a)

2. A fair is tossed repeatedly until a 'Head' appears for the first time. Let L be the number of tosses to get this first 'Head'. The entropy H(L) in bits is __________

[GATE 2014: 2 Marks]

Soln. If 1 toss is required to get first head, then probability $=\frac{1}{2}$

If 2 tosses are required to get first head then $P_2 = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

If 3 tosses are required to get first head then $P_3 = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$

Entropy

$$H = \sum_{i=1}^{n} P_i \log_2 \frac{1}{P_i}$$

$$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{16}\log_2 16$$

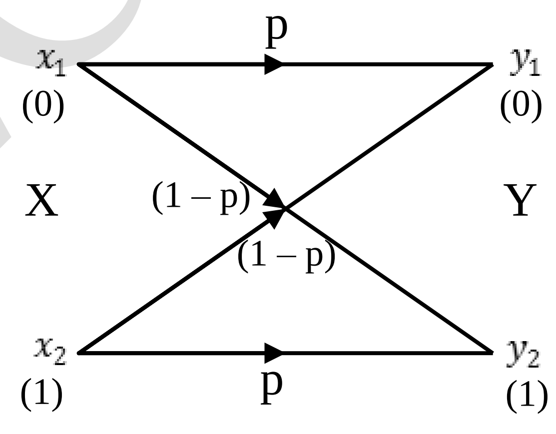
$$= \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4}$$

$$\cong 2$$

3. The capacity of a Binary Symmetric Channel (BSC) with cross-over probability 0.5 is _________

[GATE 2014: 1 Mark]

Soln.



Given cross over probability of 0.5

$\boldsymbol{P(x_1)} = {}^{\mathbf{1}}/_{\mathbf{2}}$

$\boldsymbol{P(x_2)} = {}^{\mathbf{1}}/_{\mathbf{2}}$

Channel capacity for BSC

$$\boldsymbol{(C) = log_2 n - [-\sum_{j=1}^{2} p\,({}^{y_k}/_{x})\, \log p\,({}^{y_k}/_{x})]}$$

$$\boldsymbol{log_2 2 + p \log p + (1-p)\log(1-p)}$$

$$\boldsymbol{= 1 + \frac{1}{2} log_2(1/2) + \frac{1}{2} log_2(1/2)}$$

$$\boldsymbol{= 1 - \frac{1}{2} - \frac{1}{2} = 0}$$

$\boldsymbol{C = 0}$

$\boldsymbol{Capacity = 0}$

4. In a digital communication system, transmission of successive bits through a noisy channel are assumed to be independent events with error probability p. The probability of at most one error in the transmission of an 8-bit sequence is

(a) $7(1-p)/+p/8$

(b)$(1-p)^8 + 8P(1-p)^7$

(c) $(1-p)^8 + (1-p)^7$

(d)$(1-p)^8 + p(1-p)^7$

[GATE 1988: 2 Marks]

Soln. Getting almost one error be success

Probability of at most one error = p

Say, success

Failure = 1 – p

P (X = at most 1 error)

$$\boldsymbol{= P(X=0) + P(X=1)}$$

Note that probability that event A occurs r times is given by bionomical probability man function defined as

$$\boldsymbol{P(X=r)}\ {}^{n}C_{r}\,\boldsymbol{p^{r}(1-p)^{n-r}}$$

$$= {}^{8}C_{0}\,(\boldsymbol{p})^{0}(\boldsymbol{1-p})^{8-0} + {}^{8}C_{1}\,(\boldsymbol{p})^{1}(\boldsymbol{1-p})^{8-1}$$

$$= (\boldsymbol{1-p})^{8} + \boldsymbol{8p}\,(\boldsymbol{1-p})^{7}$$

Option (b)

5. Consider a Binary Symmetric Channel (BSC) with probability of error being p. To transmit a bit say 1, we transmit a sequence of three sequence to represent 1 if at least two bits bit will be represent in error is

(a) $p^3 + 3p^2(1-p)$ (c) $(1-p)^3$
(b) p^3 (d) $p^3 + p^2(1-p)$

[GATE 2008: 2 Marks]

Soln. $\boldsymbol{P({}^{0}/_{1}) = P({}^{1}/_{0}) = p}$

$$\boldsymbol{P({}^{1}/_{1}) = P({}^{0}/_{0}) = 1 - P}$$

Reception with error means getting at the most one 1.

P(reception with error)

$$= \boldsymbol{P(X=0) + P(X=1)}$$

Using the relation of Binomial probability man function

$$\boldsymbol{P(X=r)} = {}^{n}C_{r}\ \boldsymbol{p^{r}(1-p)^{n-r}}$$

For r = 0, 1, 2,--------- n

$$= {}^{3}C_{0}\,(\boldsymbol{1-p})^{0}\boldsymbol{p^{3}} + {}^{3}C_{1}\,(\boldsymbol{1-p})^{1}\boldsymbol{p^{2}}$$

$$= \boldsymbol{p^{3} + 3p^{2}(1-p)}$$

Option (a)

6. During transmission over a certain binary communication channel, bit errors occur independently with probability p. The probability of at most one bit in error in a block of n bits is given by

(a) p^n (c) $np(1-p)^{n-1} + (1-p)^n$
(b) $1 - p^n$ (d) $1 - (1-p)^n$

[GATE 2007: 2 Marks]

Soln. Probability of at most one bit is error

P = P (non error)+P(one bit error)

Using the relation of Binomial probability man function

$= {}^{n}C_0\,(p)^0(1-p)^n + {}^{n}C_1\,(p)^1(1-p)^{n-1}$

$= (1-p)^n + np(1-p)^{n-1}$

Note, $^{n}C_0 = 1$ **and** $^{n}C_1 = n$

Option (c)

7. Let U and V be two independent and independent and identically distributed random variables such that $P(U=+1) = P\ (U=-1) = \frac{1}{2}$. The entropy H(U+V) in bits is

(a) 3/4
(b) 1
(c) 3/2
(d) $\log_2 3$

[GATE 2013: 2 Marks]

Soln. U and V are two independent and identically distributed random variables

$P(U=+1) = P(U=-1) = \frac{1}{2}$

$P(V=+1) = P(V=-1) = \frac{1}{2}$

So, random variables U and V can have following values

$U = +1, -1; \quad V = +1, -1$

$$U+V \begin{cases} -2 & When\ U = V = -1 \\ 0\ when\ U = 1,\ V = -1 & or\ U = -1, V = 1, \\ 2 & when\ U = V = 1 \end{cases}$$

$U+V = -2 \qquad P(U+V) = -2 = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

$U+V = 0 \qquad P(U+V) = 0 = \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2} = \frac{1}{2}$

$U+V = 2 \qquad P(U+V) = 2 = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

Entropy of $(U+V) = H(U+V)$

$$= \sum P(U+V)\,log_2 \frac{1}{P(U+V)}$$

$$= \frac{1}{4}log_2 4 + \frac{1}{2}log_2 2 + \frac{1}{4}log_2 4$$

$$= \frac{2}{4} + \frac{1}{2} + \frac{2}{4} = \frac{3}{2}$$

Option (c)

8. A source alphabet consists of N symbols with the probability of the first two symbols being the same. A source encoder increases the probability of the first symbol by a small amount e. After encoding, the entropy of the source

(a) increases
(b) remains the same
(c) increases only if N = 2
(d) decreases

[GATE 2012: 1 Mark]

Soln. Entropy is maximum, when symbols are equally probable, when probability changes from equal to non-equal, entropy decreases

Option (d)

9. A communication channel with AWGN operating at a signal at a signal to noise ratio SNR >>1 and bandwidth B has capacity C_1. If the SNR is doubled keeping B constant, the resulting capacity C_2 is given by

(a) $C_2 \approx 2C_1$
(b) $C_2 \approx C_1 + B$
(c) $C_2 \approx C_1 + 2B$
(d) $C_2 \approx C_1 + 0.3B$

[GATE 2009: 2 Marks]

Soln. When SNR >>1, channel capacity C

$$C_1 = B\,log_2\left(1 + \frac{S}{N}\right)$$

$$C_1 \approx B\,log_2\left(\frac{S}{N}\right)$$

When SRN is doubled

$$C \approx B\,log_2\left(\frac{2S}{N}\right) = B\,log_2 2 + B\,log_2\left(\frac{S}{N}\right)$$

$$C = B \log_2 \left(\frac{S}{N}\right) + B$$

$$= C_1 + B$$

Option (b)

10. A memoryless source emits n symbols each with a probability p. The entropy of the source as a function of n

(a) increases
(b) decreases as log n
(c) increases as n
(d) increases as n log n

[GATE 2008: 2 Marks]

Soln. Entropy H(m) for the memoryless source

$$H(m) = -\sum_{i=1}^{n} P_i \log_2 P_i \qquad bits$$

P_i = Probability of individual symbol

$$P_1 = P_2 = - - - - - - - P_n = \frac{1}{n}$$

$$H(m) = -\sum_{i=1}^{n} \frac{1}{n} \log_2 \frac{1}{n}$$

$$= \frac{1}{n} \log_2 n$$

Entropy H(m) increases as a function of $\log_2 n$

Option (a)

11. A source generates three symbols with probability 0.25, 0.25, 0.50 at a rate of 3000 symbols per second. Assuming independent generation of symbols, the most efficient source encoder would have average bit rate of

(a) 6000 bits/sec
(b) 4500 bits/sec
(c) 3000 bits/sec
(d) 1500 bits/sec

[GATE 2006: 2 Marks]

Soln. Three symbols with probability of 0.25, 0.25 and 0.50 at the rate of 3000 symbols per second.

$$\boldsymbol{Entropy\ H = 0.25\ log_2 \frac{1}{0.25} + 0.25\ log_2 \frac{1}{0.25} + 0.5\ log_2 \frac{1}{0.5}}$$

$$\boldsymbol{= 0.25 \times 2 + 0.25 \times 2 + 0.5}$$

$$\boldsymbol{= 1.5}$$

Rate of information R = r.H

R = 3000 symbol/sec

$$\boldsymbol{R = 3000 \times 1.5}$$

$$\boldsymbol{= 4500\ bits/sec}$$

Option (b)

12. An image uses 512×512 picture elements. Each of the picture elements can take any of the 8 distinguishable intensity levels. The maximum entropy in the above image will be

(a) 2097152 bits
(b) 786432 bits
(c) 648 bits
(d) 144 bits

[GATE 1990: 2 Marks]

Soln. For 8 distinguishable intensity levels

$$\boldsymbol{n = log_2 L}$$

$$\boldsymbol{n = log_2 8 = 3}$$

Maximum entropy = $\boldsymbol{512 \times 512 \times n}$

$$\boldsymbol{= 512 \times 512 \times 3}$$

$$\boldsymbol{= 786432}$$

13. A source produces 4 symbols with probability $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ *and* $\frac{1}{8}$. For this source, a practical coding scheme has an average codeword length of 2 bits/symbols. The efficiency the code is

(a) 1
(b) 7/8
(c) 1/2
(d) 1/4

[GATE 1989: 2 Marks]

Soln. Four symbol with probability $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ *and* $\frac{1}{8}$

$$Entoropy = H = -\sum_{i=1}^{n} P_i log_2(P_i)$$

$$H = -[\frac{1}{2} log_2(\frac{1}{2}) + \frac{1}{4} log_2(\frac{1}{4}) + \frac{1}{8} log_2(\frac{1}{8}) + \frac{1}{8} log_2(\frac{1}{8})]$$

$$= \frac{1}{2} + \frac{1}{4} \times 2 + \frac{1}{8} \times 3 + \frac{1}{8} \times 3$$

$$= 1 + \frac{3}{4} = \frac{7}{4}$$

Code efficiency $\frac{H}{L}$

$$= \frac{7}{4 \times 2}$$

$$= \frac{7}{8}$$

Option (b)